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By I. Ben-Gal and S. Dror, based on earlier version by F. B. Alt

Off-line **quality control** methods are often referred to the measures taken at the product and process design stages to improve product quality. G. Taguchi has developed a systematic approach to off-line quality control that has been used in Japan and has attracted attention in a number of other countries, including the United States, Germany, UK and China. What follows is a summary of main features and concepts of the so called "Taguchi Method" with emphasis on some of the associated statistical properties and some recent related developments.

Taguchi defines the quality of a product to be "the loss imparted by the product to the society from the time the product is shipped" (Phadke [1]). The emphasis is on losses caused by deviation of the product's functional characteristic (Y) from a desired target value (m). Taguchi indicates that the behavior of loss can be approximated in some instances by a quadratic function

$$L = k(Y - m)^2 \tag{1}$$

where k is a cost conversion coefficient. Note that although this function was selected conceptually, some later works justify the use of a quadratic function by approximating the unknown *loss function* with a second-degree Taylor polynomial [2]. The objective of such quality improvement method is to minimize total losses to society and practically to find a good trade-off between quality loss and product price. Taguchi's concept is different from the traditional concept of conformance to specifications (See Târcolea and Paris[3]). Thus, the mere satisfaction of specified tolerances (such as occurs with a pass/fail interpretation of loss) is less desirable than the attainment of optimal conditions. Furthermore, when considering the expected loss with respect to the functional characteristic distribution, expressed by $E[Y] = k[(\mu - m)^2 + \sigma^2]$, one can decompose the deviation from the target value into two terms, the *bias* term (measuring the distance between the mean μ and the target size m) and the variance term, σ^2, of the functional characteristic itself. Accordingly, Taguchi proposed to identify those factors that can reduce the variation in the product's functional characteristic, also called the *noise factors*, as well as those factors that can reduce the bias by shifting the mean closer to the target value, called the *signal factors*. Taguchi specified other two practical situations for the loss function: '*larger-the-better*' when the goal is to maximize Y, e.g., obtaining maximal strength, and '*smaller-the-better*' when the goal is to minimize Y, e.g., obtaining minimal weight.

Taguchi advocates a three-stage design procedure for off-line quality control: (i) *system design*; (ii) *parameter design*; and (iii) *tolerance design*.

In the system design stage, a system is designed in a somewhat traditional manner to fulfill a specific characteristic function of the product, for example, designing the main characteristics of a can opener.

In the parameter design stage, which is the key stage in Taguchi method, factors affecting the performance of Y are categorized as *controllable factors* and *noise factors*. The former are those factors that are controlled by the designer, e.g., the type of material selected to build a specific part in the product. The latter include *outer noise* (such as the variation in operational environment), *inner noise* (such as the deterioration in the product or process), and *between product noise* (such as noise due to manufacturing imperfections). The parameter design attempts to find good levels of the controllable factors, such that the effects of the noise factors on the functional characteristic are minimized, resulting in a smaller loss. In other words, Taguchi's goal is to reduce variation in the output by reducing the sensitivity of the process to the sources of variation rather than controlling these sources directly. This approach is called *robust parameter design* or simply *robust design* and exploits interaction between the causes of output variation and control factors in the process. The robust design approach got extremely popular as indicated in the section below.

In the final tolerance design stage, it may be necessary to specify narrower tolerances for some of the factors in order to fine-tune the system. This final stage is considered only if the reduction in variation achieved at the parameter design stage is insufficient.

To illustrate Taguchi's concepts, consider the following example (see Table **1**) presented by Byrne and S. Taguchi [4] in which an objective is to maximize the pull-off force of nylon tubing inserted into an elastomeric connector for use in automotive engine components. The four controllable factors are:

interference between the tubing and the connector (A), wall thickness of the connector (B), insertion depth of the tubing into the connector (C), and percent adhesive (D). Three levels were chosen for each of the controllable factors. The three noise factors are conditioning time (E), conditioning temperature (F), and conditioning relative humidity (G). Each noise factor was set at two levels (values representative of what the product would experience in the engine).

Table 1. Data from Byrne and Taguchi [4]

					Outer Array (L_8)								
					8	7	6	5	4	3	2	1	Run No.
					2	2	2	2	1	1	1	1	E
					2	2	1	1	2	2	1	1	F
					1	1	2	2	2	2	1	1	Ex F
					2	1	2	1	2	1	2	1	G
					1	2	1	2	2	1	2	1	Ex G
					1	2	2	1	1	2	2	1	Fx G
					2	1	1	2	1	2	2	1	e
Inner Array (L_9)													
Run		Factor											*S/N*
No.	A	B	C	D									Ratio
1	1	1	1	1	19.1	20.0	19.6	19.6	19.9	16.9	9.5	15.6	24.025
2	1	2	2	2	21.9	24.2	19.8	19.7	19.6	19.4	16.2	15.0	25.522
3	1	3	3	3	20.4	23.3	18.2	22.6	15.6	19.1	16.7	16.3	25.335
4	2	1	2	3	24.7	23.2	18.9	21.0	18.6	18.9	17.4	18.3	25.904
5	2	2	3	1	25.3	27.5	21.4	25.6	25.1	19.4	18.6	19.7	26.908
6	2	3	1	2	24.7	22.5	19.6	14.7	19.8	20.0	16.3	16.2	25.326
7	3	1	3	2	21.6	24.3	18.6	16.8	23.6	18.4	19.1	16.4	25.711
8	3	2	1	3	24.4	23.2	19.6	17.8	16.8	15.1	15.6	14.2	24.832
9	3	3	2	1	28.6	22.6	22.7	23.1	17.3	19.3	19.9	16.1	26.152

The design that Taguchi recommends is actually the product of two designs. A so-called *inner array* is constructed to study the effects of the controllable factors themselves, and for each cell of this inner array, a design for the noise factors is run, called the *outer array*. The mean and variance of the functional characteristic over the outer array are computed for each cell of the inner array and then analyzed with respect to the controllable factors. Taguchi himself recommends that orthogonal arrays (*See* **Orthogonal Arrays and Applications**; **Orthogonal Arrays**) be used for both the inner and outer arrays, but this choice has been criticized on the grounds that it ignores the possibility of important interaction effects between the controllable factors. (See Hunter[5]). Also, it is at least questionable whether a highly systematic arrangement for the outer array such as a fractional factorial design (*See* **Fractional Factorial Designs, issues in**; **Fractional Factorial Designs**), in which not all the factors' level are experimented and analyzed, can be expected to reflect accurately the true behavior of the noise factors, which behavior is by definition unsystematic and possibly random. [6] [7]

In order to facilitate the study of the variation (noise) in the response as well as the mean response (signal) for each row of the inner array, Taguchi introduces a *signal-to-noise index*. Kackar [6] provides an overview of these indices and demonstrates how they are related to the expected loss function as well as to the more conventional mean-square-error measure. For the Byrne and Taguchi example, the appropriate index is

$$S/N= -10\log\sum_{i=1}^{n_2}\frac{y_i^{-2}}{n_2}$$

where the values are shown in Table 1 with, $n_2 = 8$.

Although a formal **analysis of variance** can be conducted to determine which factors are significant, Taguchi advocates graphical methods that are simpler to adopt by the industry (as indeed actually happened). For each controllable factor, this would include plots of the average *S/N* ratios for each level, as well as plots of the mean response. Such an analysis for the Byrne and Taguchi example yielded the following choice of levels for the controllable factors: A_2 (medium interference), B_2 (medium wall thickness), C_3 (deep insertion), and D_1 (low percent adhesive). However, cost considerations resulted in using B_1 (thin wall thickness) in place of B_2.

After the operating conditions are determined using off-line quality control, it is necessary to follow this up with those quality control activities needed during manufacturing. Taguchi refers to these as on-line activities which include (i) diagnosis and adjustment of process; (ii) forecasting and correction; (iii) measurement (inspection) and disposition; and (iv) after service by the sales department. For additional detail, refer to Taguchi [8]. G. Taguchi [9] has also developed *accumulation analysis*, a technique for testing independence in ordered categorical data. This procedure is reviewed by Nair [10].

Taguchi has been enthusiastically praised for emphasizing the use of designed experiments not only to set a product's characteristics at target values but also to reduce variation around these targets, as well as for putting forward a dedicated strategy for effecting these goals. Moreover, the straightforward, systematic nature of Taguchi's program makes it relatively easy to introduce to experimenters and practitioners and, hence, easy to implement. On the other hand, certain important statistical details of Taguchi's proposed strategy— e.g., the selection of experimental designs, the method of analysis, the sequential experimental steps and other— have been heavily criticized as simplistic and inaccurate. The fact that Taguchi's orthogonal arrays make it impossible to sort out possibly important interaction effects is one example. The relative lower resolution level of Taguchi's fractionally factorial arrays is another example, where the sequential designs of the *response surface methodology* (RSM) has been shown to require far fewer experimental runs than would a sequence of Taguchi's designs [7]. A deep discussion of the pros and cons of off-line quality control can be found in the article by Kackar [6] and the ensuing discussions by Box, Easterling, Freund, Lucas, and Pignatiello and Ramberg, as well as the article by Hunter [5].

In the last decades, the development of Taguchi's robust design approach continued to prosper despite the ambiguous ideas on his method. Many such developments either focus on new applications to the Taguchi method or aim to couple and strengthen it by using new analytical tools from area such as machine learning, data mining and information theory. For example, Kumar and Motwani [11] extended the applicability of the Taguchi methods for process optimization - from manufacturing to a service setting. Using a real-world example pertaining to the complaint correction process of a small export company, it was demonstrated that the Taguchi's robust experimental design, hitherto employed to optimize product specifications and process parameters in manufacturing settings, can be employed, with equal effectiveness, to optimize the factors that influence a service process. Another real world application was suggested in Ben-Gal et al. [12] that demonstrated how the Taguchi method can be used to obtain a robust eco-design (ecological design), while using the highly non-linear models that represent physical behavior of ecological noise factors.

Lunani et al. [13] demonstrated the limitation of the data analysis methods recommended by Taguchi. Two graphical methods were proposed: the sensitivity-standard deviation plot and the gamma-plot. These graphical methods are related to the mean-variance plot and the lambda-plot that have been found useful in analyzing data from static robust design studies. Chao-Ton [14] proposed a procedure on the basis of principal component analysis (PCA) to optimize the multi-response problems in the Taguchi method. With the PCA, a set of original responses can be transformed into a set of uncorrelated components. Therefore, the conflict for determining the optimal settings of the design parameters for the multi-response problems can be reduced. Chao-Ton and Kun-Lin [15] presented a novel means of applying neural networks to Taguchi's dynamic problem. The neural network utilizes the feature hid in the designed experiment not only to learn, but also to store the feature into the connected weights between layers. The connected weights reach steady-state after neural learning. Kenett & Zacks [2] illustrated how to approximate the expected value and the variance of a known nonlinear response by using a Taylor series. They found the robust solution analytically, and compared it to a solution found by a numerical Monte-Carlo sampling. Chipman [16] used Bayesian methods for fitting the response models and their subsequent optimization by incorporation of reliable assessments of uncertainty into the analysis of robust design experiments. Jayaram and Ibrahim [17] presented a method of achieving high yield and robust design for multiple responses, utilizing the Cp and Cpk capability indices implemented in on-line quality control techniques. The proposed method is applied to a single response problem and two multiple response problems. The results showed that the proposed method is capable of producing good manufacturing yield and robust design simultaneously. Nair and Taam [18] considered general and flexible methods for analyzing location and dispersion effects and use three real applications to illustrate the methods. Two applications demonstrate the usefulness of functional regression techniques for location and dispersion analysis while the third illustrates a parametric analysis with two-stage modeling. Both a mean-variance analysis for random selection of noise settings as well as a control-by-noise interaction analysis for explicitly controlled noise factors are considered. Kowalski [19] used a split-plot fractional factorial design and proposed a method for constructing sixteen-run experiments. Semi folding was also used to add eight more runs. The resulting 24 run design breaks some of the alias chains and provides some degrees of freedom for estimating a subplot error variance. Also, an alternative 24 run design based on the balanced incomplete block design was proposed. Romano [20] presented a general framework for the multivariate problem. Within the framework, both parameter and tolerance design are handled in an integrated manner. The used optimization criterion is based on a single value in terms of the quadratic loss function, and it is selected in order to incorporate both statistical information and economic information relevant to the product or process. McLeod [21] explored blocked fractional-factorial split-plot designs for robust parameter design. A ranking scheme for such designs was developed and, using a search algorithm, a catalog of 32-run optimal designs was provided. Dasgupta [22] developed a parameter design methodology in the presence of feedback control for long-duration processes. Systems that follow a pure-gain dynamic model were considered and the best proportional-integral and minimum mean squared error control strategies were developed using robust parameter design. Chang [23] proposed an alternative approach based on data mining tools to model and optimize dynamic robust design with missing data. First, a back-propagation network is trained to construct the response model of a dynamic system. Second, three formulas of performance measures are developed to evaluate the predicted responses of the model. Finally, a genetic algorithm is performed to obtain the optimal parameter combination via the response model. Frey [24] proposed an approach for evaluating the effectiveness of robust parameter design methods. A hierarchical probability model was presented that enables an investigator to represent assumptions about regularities in system responses such as effect sparsity, hierarchy, and inheritance. Kang and Roshan [25] argued that hierarchy principle should not be altered for achieving the robustness objective of the experiment. The authors proposed a Bayesian approach to develop single arrays which incorporate the importance of control-by-noise interactions without altering the effect hierarchy. Kovach [26] proposed a robust design method for multiple quality characteristics where the goal is to first reduce the variability of the system under investigation and then attempting to locate the mean at the desired target value. The paper investigated the use of a response surface approach and a sequential optimization strategy to create a flexible and structured method for modeling multi-response problems in the context of robust design. Nonlinear programming was used as an optimization tool. Besseris [27] presented an additive ranking scheme based on converting the responses of interest to rank variables regardless of the nature of each response and the optimization direction that may be issued for each of them. Collapsing all ranked responses to a single rank response allows simultaneous optimization for all the considered factor settings. Dasgupta [28] developed an integrated approach for estimation and reduction of measurement variation through a single parameter design experiment. Systems with a linear signal-response relationship are primarily addressed in this work. The noise factors are classified into a few distinct categories based on their impact on the measurement system. A random coefficients model that accounts for the effect of control factors and each category of noise factors on the signal-response relationship is proposed. A suitable performance measure is developed using this general model, and conditions under which it reduces to the usual dynamic signal-to-noise ratio are discussed. Two different data analysis strategies, *response function modeling* and *performance measure modeling*, for modeling and optimization were proposed and compared. The effectiveness of the proposed method has been demonstrated via a simulation study using the Taguchi's drive-shaft experiment. Ben-Gal [29] suggested an approach that uses data compression measures, such as the Entropy, and Huffman Coding, to assess the effects of noise factors on the reliability of tested systems. He extends the Taguchi method for robust design by computing the entropy of the percent contribution values of the noise factors and show by implementing the proposed approach the designer obtains the minimal expected number of steps to find the disturbing noise factor. Hossein et al. [30] compared two classes of plans that they call *desensitization and robustness experiments*. With a desensitization experiment, one needs knowledge of a dominant cause and the ability to set its level in the experiment. With a robustness experiment, one uses time or location to indirectly generate the effect of the dominant causes of the output variation. The authors explored qualitatively and quantitatively the differences between robustness and desensitization experiments. They argue that for an existing process, desensitization is the preferred choice. Gremyr [31] investigated whether a design of experiments application in a company that works with robust design management (RDM) reflects the principles and practices of RDM. The findings of this paper were based on an empirical study of a medium-sized Swedish enterprise that develops and manufactures consumer products for domestic use and actively uses RDM. The results show the difficulties that companies face when trying to introduce RDM principles. The research also provides insights into RDM implementation work at the company in question. Timothy [32] presented a Bayesian approach to process optimization for a general class of robust parameter design models, including both normal and non-normal responses, in the split-plot context using an empirical approximation of the posterior distribution for an objective function of interest. Jeang [33] developed a new process capability index (PCI) to reflect the differences among alternative designs for a better decision making at the product design and process planning stages. However, using such a deterministic approach with a new process capability index has been shown to be disadvantageous when dealing with uncertainties during the product design and process planning activities.

In summary, although some of the statistical aspects of the Taguchi methods are arguable, there is no dispute that they are widely applied to various processes until these days. A quick search in the world wide web, reveals that the method is being successfully implemented in diverse areas, such as the design of VLSI, optimization of communication networks, development of electronic circuits, laser engraving of photo masks, cash-flow optimization in banking, runway utilization improvement in airports and even government policymaking.

Related Articles and Terms

Design of Experiments: Industrial and Scientific Applications; Fractional Factorial Designs; Orthogonal Arrays and Applications; and Quality Control, Statistical. Response Surface Methodology; Principal Component Analysis; Entropy; Split-Plot; Process Capability Index.

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