On the Use of Data Compression Measures to Analyze Robust Designs

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***Abstract*—In this paper, we suggest a potential use of data compression measures, such as the Entropy, and the Huffman Coding, to assess the effects of noise factors on the reliability of tested systems. In particular, we extend the Taguchi method for robust design by computing the entropy of the *percent contribution* values of the *noise factors*. The new measures are computed already at the parameter-design stage, and together with the traditional S/N ratios enable the specification of a robust design. Assuming that (some of) the noise factors should be naturalized, the entropy of a design reflects the potential efforts that will be required in the tolerance-design stage to reach a more reliable system. Using a small example, we illustrate the contribution of the new measure that might alter the designer decision in comparison with the traditional Taguchi method, and ultimately obtain a system with a lower quality loss.**

Assuming that the percent contribution values can reflect the probability of a noise factor to trigger a disturbance in the system response, a series of probabilistic algorithms can be applied to the robust design problem. We focus on the Huffman coding algorithm, and show how to implement this algorithm such that the designer obtains the minimal expected number of tests in order to find the disturbing noise factor. The entropy measure, in this case, provides the lower bound on the algorithm's performance.

***Index Terms*—Compression rate, control & noise factors, entropy, experimentation, performance measure, robust designs, S/N ratio, Taguchi method.**

ACRONYMS[1]

S/N	signal to noise
ANOVA	analysis of variance
df	degree of freedom
PC	percent contribution
PMF	probability mass function
StdDev	standard deviation

NOTATIONS

Y	system response
S^2	sample variance of the system response
$L(Y)$	quality loss function
K	quality loss coefficient
t	required target value
$\mathbf{x}$	vector of control factors
$\mathrm{SS_x}$	sum of squares of factor x
v_x	degree of freedom of factor x
V_x	variance associated with factor x
V_e	mean-square-error
PC_x	percent contribution of factor x
D_i	design i, $i = 1, 2, \ldots$
$H(\mathrm{D}_i)$	entropy of the ith design
L	expected number of tests

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[1]The singular and plural of an acronym are always spelled the same.

I. INTRODUCTION

A. Literature Review

THE MAIN objective in the Taguchi method [1]–[3], is to design robust systems which are reliable under uncontrollable conditions. The method aims to adjust the design parameters (known as the *control factors*) to their optimal levels, so that the system response is robust; that is, the system response is insensitive to noise factors, which are hard or impossible to control [3]. Although some of the statistical aspects of the Taguchi methods are disputable (eg, [4]–[6]), there is no dispute that they are widely applied to various processes. A quick search in related journals, as well as the World Wide Web, reveals that the method is being successfully implemented in diverse areas, such as the design of VLSI; optimization of communication & information networks, development of electronic circuits, laser engraving of photo masks, cash-flow optimization in banking, government policymaking, and runway utilization improvement in airports [3], [7]–[9].

The Taguchi method has been extensively elaborated & analyzed in published papers. Box & Meyer [10] suggested a method to estimate the variance of the response, and identified factors that affect it with small nonreplicated designs. Leon [5] introduced the concept of *PerMIA*, a performance measure independent of adjustments. Their measure was suggested as a replacement for Taguchi's S/N ratios during the analysis stage. Box [11] criticized the statistical tools used by Taguchi, and suggested working with two ratios based on the response mean & variance, independently. He also introduced the *Lambda Plot* as an efficient tool to obtain a compatible transformation. Pignatiello [12] considered multiple quality characteristics, and introduced priority-based approaches to be used in such cases. Steinberg [13] mentioned important statistical considerations that should be carefully addressed before implementing the robust design method. McCaskey & Tsui [14] developed a two-step robust design procedure for *dynamic systems* whose target value depends also on an input signal that is set by the system's operator. Kenett & Zacks [15] illustrated how to approximate the expected value & the variance of a known nonlinear response by using a Taylor series. Consequently, they found the robust solution analytically, and compared it to a solution found by a numerical Monte-Carlo sampling. Tsui [16] investigated the response surface model (RSM) approach, and compared it to the Taguchi method for a dynamic robust design. Sanchez [17] considered a framework for robust design using simulation tools. Following the above extensions to the Taguchi method, in this paper we suggest new entropy-based performance measures.

B. The Taguchi Method and the Proposed Approach

Taguchi characterized three types of quality loss functions for a given system [1]–[3]:

i) *nominal-the-best* is where the designer wants to achieve a particular target value, for example, a required output voltage of a circuit.
ii) *smaller-the-better* is where the designer wants to minimize the system response because quality decrease as the system response increases. Some examples are the response time of a computer, or a radiation leakage from a microwave oven.
iii) *larger-the-better* is where the designer wants to maximize the system response since quality increases with the system response. For example, the bond strength of a weld point.

In this paper, we focus on the *nominal-the-best* quality loss. However, the suggested method can be well applied to all other types of quality loss. In fact, our objective, which is to find a system with a low entropy measure, does not depend on the type of quality loss.

Taguchi's motivation for continuous improvement emerged from his definition of the quality loss function. Because the nominal-the-best function implies a higher rate of quality loss as the system response gets far from a required target, a quadratic function was selected as the simplest mathematical function that preserves the desired behavior. The quality loss $L(Y)$ is given by

$$L(Y) = K(Y - t)^2, \tag{1}$$

where Y is the system response, K is a cost constant called the *quality loss coefficient*, and t is the required target. The response of a system, and, as a result, its quality characteristics, are influenced by three types of factors:

i) *signal factors* that are set by the operator of the system in later stages of the product life;
ii) *control factors*, $\mathbf{x}$, that are set by the designer of the system; and
iii) *noise factors*, that cannot be directly controlled by neither the designer nor the operator.

Both control & noise factors can take multiple values called *levels*.

Control factors are those design parameters that can be freely specified by the designer. Taguchi [2], [3] divided these control factors into two subsets, $\mathbf{c_1}$ & $\mathbf{c_2}$. Belonging to $\mathbf{c_1}$ are those factors influencing both the response mean, and the response variance. Belonging to $\mathbf{c_2}$ are those factors influencing only the response mean. Taguchi used these subsets to obtain the robust designs in a two-stage procedure, which is explained below. Later robust design approaches further divided the control factors into four subsets, depending on their influence on the mean, and on the variance, as well as based on economic considerations [4], [12], [18], [19].

Noise factors were usually classified by Taguchi into three classes [2], [3]:

i) *external noise factors* that typically describe the environmental conditions, such as temperature, dust, humidity etc.;
ii) *unit-to-unit variation* that typically addresses the inevitable variations in a manufacturing process; and
iii) *deterioration* that typically refers to the deterioration in functional characteristics of sold products as time passes.

Taguchi's main idea was to control the noise factors *indirectly* by examining how they are affected by different settings of the control factors. He suggested analyzing the joint effects of control & noise factors, and for this purpose, proposed a performance criterion called *signal-to-noise ratio* (S/N). The S/N ratio for the nominal-the-best loss in the case where the response variance is related to the response mean is [3]

$$\text{S/N} = 10 \log_{10} \left(\frac{\overline{Y}^2}{S^2} \right), \tag{2}$$

where $\overline{Y}$ is the average response, and S^2 is the variance of the response over various experimented samples of designs. Taguchi's objective was to design a system such as to maximize the S/N value while keeping the response on the target. In particular, his method for robust design was divided into two stages called *parameter-design*, and *tolerance design*.

In the *parameter-design* stage, the designer has to determine the best setting of the control factors to minimize the quality loss. This objective is achieved by a two-step procedure [2], [3]. First, set those control factors in $\mathbf{c_1}$ to maximize the S/N ratios in order to minimize the sensitivity of the response to noise. Second, use the factors in $\mathbf{c_2}$ to adjust the mean response to the desired target, based on the assumption that the response mean can be altered independently from the response variance due to these tuning factors. The underlying assumption at this stage is that the setting of control factors does not affect the manufacturing costs. During parameter design, Taguchi assumed a wide tolerance on the noise factors; and, under these conditions, tried to minimize the sensitivity to noise. If at the end of the parameter-design stage the quality loss has to be further reduced, which is the situation in most practical applications [3], the designer has to continue to the *tolerance design* stage. Our proposed approach is useful only if the tolerance of (some of) the noise factors can be reduced (naturalized) during the tolerance design stage.

In the *tolerance design* stage, the designer selectively reduces the tolerances of the noise factors to further minimize the quality loss. A trade-off is often considered between the reduction in the quality loss, and the costs required to reduce the tolerances of noise factors. Taguchi suggested to perform the tolerance design stage only *after* the S/N-based optimal design has been selected in the parameter-design stage [2], [3]. Otherwise, it was claimed that the costly tolerance design will be somewhat "wasted" on

TABLE I
ANOVA AND PCONT CALCULATIONS

	An L_8 Outer Array with 3 noise factors								Noise factor		
	1	1	1	1	2	2	2	2	A		
	1	1	2	2	1	1	2	2	B		
	1	2	1	2	1	2	1	2	C		
Inner Array	Experimental Result (control × noise effects)								**Mean**	**S/N**	**H**
D_1	20.39	17.57	17.54	13.03	26.69	22.85	23.37	18.96	20.05	13.49	1.527 (76%)
D_2	16.76	16.43	15.32	16.28	24.62	24.28	23.44	23.70	20.10	13.58	0.143 (7%)
D_3	20.87	19.53	17.86	15.36	24.91	22.54	20.47	18.82	20.05	16.76	1.514 (76%)
D_4	15.52	24.44	8.27	16.06	23.22	32.44	15.49	24.37	19.98	8.47	1.60 (80%)

a nonoptimized system, leading to a higher investment in tolerance reduction to achieve the desired low quality loss. Thus, note that Taguchi's approach optimizes the system in each of the two design stages *independently*. It does not consider a possible situation where a nonoptimal system in the parameter-design stage achieves the lowest tolerance-reduction cost in the tolerance design stage. Here we aim to address such situations.

In this paper, we provide a measure of the expected tolerance-reduction efforts already at the parameter-design stage. Consequently, instead of optimizing the system at the parameter-design stage according to the S/N ratios, and then carrying on with the S/N-based optimal system to the tolerance design stage, we propose a more unified approach for the design. We suggest assessing in advance the number & effects of noise factors which will have to be naturalized in the tolerance stage. We assume that the cost of tolerance minimization is proportional to the number & effects of those noise factors that should be naturalized in the tolerance design stage. Accordingly, in addition to the S/N measure, we suggest a new entropy-based measure for each design configuration, denoted by H(design). Simultaneous inspection of both measures helps the designer to identify a system configuration whose *overall cost following both design stages* is low. Practically, we claim that in the parameter-design stage it might be better to select a design configuration with a slightly higher S/N ratio, yet a lower H value that requires relatively lower efforts in the tolerance design stage. We now follow with the description of how to obtain the proposed entropy measure.

II. Compressibility of the Percent Contribution Values

Central to Taguchi's parameter design stage is the implementation of designed experiments. The experiments are conducted to identify the effects of the control factors on the system response under various settings of the noise factors. In particular, the method suggests the use of a *crossed array*, which is a product of dual experimental arrays: an *inner array* which determines the levels of the controllable factors, and an *outer array* which indicates the levels of the noise factors, as they are controlled during the experiment. The outer array deliberately introduces a systematic noise during the experiment to identify the design configurations that are less sensitive to the noise [1], [3]. Table I exemplifies such a crossed array with four system configurations in the inner array, and eight combinations of noise factors in the outer array. The experiment in the table is explained later.

Once the experiments are executed, the Analysis of Variance (ANOVA) is used to partition the total response variability into components associated with different control factors, and to identify the significant ones. The signal-to-noise (S/N) measures are then computed for each row in the inner array. These S/N measures reflect the reliability of the various experimental configurations under the systematic noise, which is imposed by the outer array. Nonetheless, the S/N measures overlook the *individual contribution of each noise factor* to the variability in the response. Knowledge regarding the spread of the noise effects is important when the designer has to determine the required efforts to further optimize the system. For example, it might be crucial for the designer to know whether the system is sensitive to only one of the noise factors, or whether it is sensitive to all of them. Thus, although the Taguchi method is focused on the design of reliable systems, it cannot distinguish at the parameter-design stage between two design configurations having similar S/N ratios that are obtained from a totally different divergence of noise effects. Next, we show how to obtain the new entropy measures that can assist in bridging this gap.

The *percent contribution* measures were used by Taguchi for the interpretation of experimental results [3], [7]. The percent contribution values reflect the relative portion of the total variation observed in an experiment which is attributed to each factor. It is a function of the sums-of-squares, $\mathrm{SS}x$, for each factor x, indicating its relative power to reduce the response variation. In other words, the percent contribution of a given factor indicates the potential reduction in the total variation that can be achieved, if this factor is controlled precisely. Taguchi distinguished between V_x, the variance associated with factor x *including* the experimental noise (error), and V_x', the variance associated purely with this factor, reflecting the contribution to variability only due to changes in the factor levels. Accordingly, the "pure" variance which is associated with factor x is given by

$$V_x' = V_x - V_e,$$

where V_e is the mean-square-error. Equivalently, in terms of the sum-of-squares

$$\frac{SS'_x}{v_x} = \frac{SS_x}{v_x} - V_e, \tag{3}$$

where v_x denotes the degree of freedom (df) for the factor, and SS'_x are the sum-of-squares associated 'purely' with the factor levels variation. The *percent contribution* of factor x, denoted here by PC_x, is obtained by the ratio between the 'pure' sum-of-squares of the factor, and the total sum-of-squares, SS_T. Based on (3) the percentage is computed as [7]

$$\mathrm{PC}_x = 100 \cdot \frac{SS'_x}{SS_T} = 100 \cdot \frac{SS_x - V_e \cdot v_x}{SS_T}. \tag{4}$$

Note that the total percent contribution due to all factors, and due to the error term, must add up to 100%. When using the sum-of squares rather than the pure ones, a closely related measure is obtained, which is known as the *eta-squared*. In the parameter-design stage, Taguchi measures the percent contribution of the controllable factors, and uses a rule-of-the-thumb that the sum of the percent contributions due to all significant factors should be larger than 85% [7]. Such a threshold assures that no significant controllable factor was forgotten & omitted from the experiment. In the tolerance design stage, Taguchi uses the percent contribution to see that the contribution of noise factors to the response variance exhibits the typical Pareto principle [3]. Hence, Taguchi uses the percent contributions measure as an estimation of the experiment *adequacy*. We suggest measuring the *entropy* of the percent contributions of the noise factors in the outer array. The reason is twofold: first, to measure the combined contribution of the noise factors to the response variation in the parameter-design stage, and second, to assess the potential reliability of the system, if some noise factors can be naturalized in the tolerance-design stage. Our choice of the entropy measure is explained next.

The entropy of the PC_x values provide a rational measure to assess the contribution of different noise factors to the *variability*, or alternately, to the *uncertainty* in the system output. In general, the entropy is an ultimate measure of the average uncertainty in a random variable (r.v.) Λ with probability mass function $p(\lambda)$, and is given by [20]

$$H(\Lambda) = -\sum p(\lambda) \log_2 p(\lambda). \tag{5}$$

When the logarithm is base 2, the entropy is measured in *bits*. Note that the entropy is a function of the distribution of the r.v., and does not depend on its actual values. It is a concave function of the distribution, and equals zero only in the deterministic case, i.e., when the probability of one of the values is equal to one. Moreover, it obtains a positive value within the bounds

$$0 \leq H(\Lambda) \leq \log_2 q, \tag{6}$$

where $q = |\Lambda|$, i.e., the number of elements in the range of Λ [20], [21]. The upper bound is obtained i.f.f. Λ has a uniform distribution over the range. Shannon [20] used the above properties, and showed that the entropy function provides a lower bound on the expected description length of the random variable.

TABLE II
ANOVA AND PERCENT CONTRIBUTION (PC_x) VALUES OF NOISE EFFECTS FOR SYSTEM D_3

Factor x	SSx	df	MSx	F	P-value	PC_x (%)
Model	58.654	3	19.551	132.53	0.0002	–
A	21.517	1	21.517	145.85	0.0003	36.1
B	29.414	1	29.414	199.39	0.0001	49.4
C	7.722	1	7.722	52.35	0.0019	12.8
error	0.590	4	0.1475			1.7
Total	59.244	7				100%

We apply the entropy function to each design configuration, i.e., calculating the entropy of the percent contribution values of all the factors as obtained in the i^{th} design

$$H(\mathrm{D}_i) = -\sum_x \mathrm{PC}_x \log_2 \mathrm{PC}_x. \tag{7}$$

Thus, we adopt a "probabilistic approach" with regard to the contribution of each noise factor in each design to the response's uncertainty. The advantages in applying the entropy to the percent contribution values can be demonstrated through the following numerical experiment.

Table I presents a crossed array that exemplifies the contribution of the suggested performance measure. The experiment investigates the effects of various controllable factors (that are omitted for simplicity of presentation), as appeared in the inner array that consists of four tested systems (designs): D_1–D_4. The L_8 outer array shows the setting of three noise factors: A, B, and C (noise factors' interactions are ignored in this example). Each entry in the crossed array presents the response of a tested system under the specific setting of the noise factors. The last three columns correspond to three performance measures (the first two are traditionally proposed by the Taguchi method): *i*) *Mean* is the mean of the response; *ii*) *S/N* is the signal-to-noise ratio; and *iii*) H is our suggested measure, the entropy of the percent contribution values of the noise factors given in (7), including (in parentheses) the respective percentage from the entropy upper-bound given in (6). The percentage of the upper bound is informative because the entropy function has a nonlinear rate in the distribution: it increases steeply for low values (the almost "deterministic" cases), and is rather flat around its maximal value. Thus, this percentage scales the differences between designs that are affected by a different number of noise factors, relative to designs that differ one from another only due to their PC_x distributions.

As explained above, the entropy measure for each design is obtained from the ANOVA of its responses with respect to the outer array. For example, the entropy measure of D_3, the system with highest S/N value, is calculated by ANOVA of D_3's eight responses, as presented by columns 1–6 in Table II. Column 7 presents the percent contribution values calculated according to (4). For example, considering factor A, $\mathrm{PC}_\mathrm{A} = 100 \cdot ((21.517 - 0.1475 \times 1)/59.244) = 0.361$; thus, 36.1% of the response variance is due to noise factor

A, and could potentially be reduced if this factor could be naturalized (eg, by adding a cooling unit to the system, if factor A represents the environment temperature). Based on all the percent contribution values, the designer uses (7) to compute the entropy for each design. In this example, $\mathrm{H(D_3)} = -0.361\log_2 0.361 - 0.494\log_2 0.494 - 0.128\log_2 0.128 - 0.017\log_2 0.017 = 1.514$ bits. This value is equal to 76% of the entropy upper bound for a system with $q = 4$ noise & error factors: $\log_2 4 = 2$ bits.

The entropy measures of the PC_x values for each system are presented in the last column of Table I, and reflect the contribution of various noise & error factors to the response variability. As the entropy increases, more noise factors have to be naturalized in order to obtain a more reliable system. If a single factor contributes most of the variability, the entropy of the PC_x measures will be close to zero, whereas, if the variability contribution is spread equally among the factors, the entropy increases up to log(number of factors), as indicated in (6). Measuring various designs by their entropy measures thus provides an efficient way to assess the future efforts in the tolerance design stage to achieve a more reliable system that is independent of the design area of specification. Note, on the other hand, that the entropy value of a system, similar to its S/N value, is a *relative* measure, which is mainly used for the comparison with other systems. There is no clear-cut decision rule for selecting a system which is solely based on its entropy value. A roughly derived "rule of thumb" for a system selection, which based on the entropy measure, is given in Appendix I. There we rely on the Pareto principle, and find the q-ary entropy bound (which is equivalent to the percentage of the upper bound in (6)) of those systems where 20% of the noise factors evenly contribute 80% of the response variability. The derived entropy bound depends only on the number of noise factors, and gives a point of reference to indicate whether a system is a good candidate for the tolerance-design stage. In particular, the entropy bound for systems with 4 noise & error factors, as those systems considered in Table I, is equal to 40%. Such a bound implies that any system for which the percentage of the upper bound is lower than 40% (in this example it applies only to system $\mathrm{D_2}$ with a percentage of 7%) can be considered as a good candidate for the tolerance design stage. Once the design candidates are identified, the designer can sort the factors according to their $\mathrm{PC_x}$ values, and apply the tolerance design to the factors that contribute most (e.g., 80%) to the response variability.

Note that the use of the entropy performance measure can alter the designer's selection in comparison with the traditional Taguchi method. In this example, although $\mathrm{D_3}$ would traditionally be chosen according to its highest S/N value (note that the mean values for all designs are almost identical in this example), its relatively high entropy measure indicates that the response variability is affected by all noise factors. Such a spread of noise effects increases the probability of a noisier response which is less reliable. Even if the nosiest factor will be naturalized in the tolerance design stage, the resulting system will still be affected by the other noise factors. On the other hand, although the S/N value of $\mathrm{D_2}$ is lower than the S/N of $\mathrm{D_3}$, i.e., 13.58 vs. 16.76, its lower entropy measure clearly indicates that most of the variability in the response is associated with a single noise factor,

TABLE III
ANOVA AND PERCENT CONTRIBUTION (PC_x) VALUES OF NOISE EFFECTS FOR SYSTEM $\mathrm{D_2}$

Factor x	SSx	df	MSx	F	P-value	PC_x (%)
Model	123.511	3	41.170	287.58	< 0.0001	–
A	122.070	1	122.070	852.67	< 0.0001	98.3
B	1.403	1	1.403	9.80	0.0352	1.0
C	0.038	1	0.038	0.26	0.6344	– 0.1
error	0.573	4	0.143			0.8
Total	124.084	7				100%

factor A in this case, with $\mathrm{PC_A} = 98.3\%$, as can be seen in Table III. Note that the $\mathrm{PC_x}$ value of factor C is negative because the factor is insignificant. One can add factor C to the error term & rerun the analysis; however, this will result in a similar entropy value which is practically identical to the one presented in Table I. To conclude, it might be wiser to select $\mathrm{D_2}$ at the parameter design stage if it is known that factor A can be naturalized later. The other two designs can be discarded based on their joint S/N & H measures.

Further analysis for the comparison of systems $\mathrm{D_2}$ & $\mathrm{D_3}$ is available by using the empirical models that are obtained for each system. These empirical models can be used to predict the potential S/N ratio once the most influential noise factor (in each system) is naturalized. In particular, based on the experimental results in Table I, the empirical response models for both systems (with the coded factors -1, 1) are, respectively

$$\begin{aligned} Y(D_2) &= 20.104 + 3.905\cdot\mathrm{A} - 0.420\cdot\mathrm{B} + 0.067\cdot\mathrm{C} + \varepsilon \\ Y(D_3) &= 20.045 + 1.640\cdot\mathrm{A} - 1.918\cdot\mathrm{B} - 0.983\cdot\mathrm{C} + \varepsilon. \end{aligned} \tag{8}$$

Note from Tables II & III that both these models are statistically significant, and obtain a low p-value. Thus, the designer can use these models to predict quite accurately the responses of the system under various noise-factor configurations. These values enable the estimation of the response mean, the response variance, and thus, the potential S/N ratio & H values. Based on the above empirical models, the predicted values, means, and S/N ratios for both systems are practically identical to those calculated from the experimental observations, as seen in Table IV (columns 3 vs. 2, and 6 vs. 5). Note that the predicted H values are lower than the experimental ones because the noise terms of the former are equal to zero. Under the existing circumstances, the designer can use the empirical model to predict the potential S/N ratio in case the most influential noise factor is naturalized. This is done by simply deleting the relevant term in the empirical response model, and regenerating the responses for all the noise factor combinations in the outer array. Table IV presents the predicted responses of system $\mathrm{D_2}$ with factor A being naturalized (column 4), and system $\mathrm{D_3}$ with factor B being naturalized (column 7); and compares both to the original responses in Table I (columns 2 & 5 respectively). Note that in case of tolerance elimination, system $\mathrm{D_2}$ obtains a much higher S/N ratio compared to that of system $\mathrm{D_3}$. Namely, the potential increase in the S/N ratio for system $\mathrm{D_2}$ is from 13.58 to 32.90, while the potential increase in the S/N ratio for system $\mathrm{D_3}$ is from 16.76 only to 19.83. This result emphasizes once again that under the assumption that the noise factors can be effectively naturalized, and under relevant cost-benefit considerations, it might be wiser

TABLE IV
ACTUAL AND ESTIMATED RESPONSES; AND S/N RATIOS FOR SYSTEMS D_2 & D_3, UNDER VARIOUS NOISE-FACTOR SETTINGS

Systems	System D_2			System D_3		
Noise factors settings	exp. values	predicted values	Predicted values: factor A naturalized	exp. values	predicted values	predicted values: factor B naturalized
111	16.76	16.55	20.46	20.87	21.31	19.39
112	16.43	16.69	20.59	19.53	19.34	17.42
121	15.32	15.71	19.62	17.86	17.47	19.39
122	16.28	15.85	19.75	15.36	15.50	17.42
211	24.62	24.36	20.46	24.91	24.59	22.67
212	24.28	24.50	20.59	22.54	22.62	20.70
221	23.44	23.52	19.62	20.47	20.75	22.67
222	23.70	23.66	19.75	18.82	18.78	20.70
Mean	20.10	20.10	20.10	20.05	20.05	20.05
StdDev	4.21	4.20	0.46	2.91	2.90	2.04
S/N	13.58	13.60	32.90	16.76	16.80	19.83
H	0.143 (7%)	0.094 (5%)	0.160 (8%)	1.514 (76%)	1.416 (71%)	0.834 (42%)

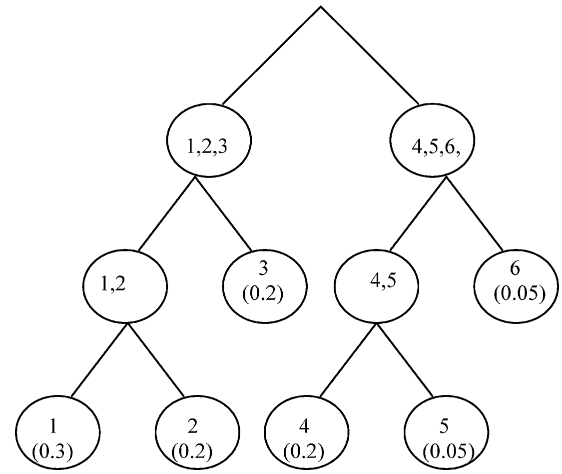

Fig. 1. A binary test tree with six noise factors, and an average number of $L = 2.75$ tests.

in the parameter design stage to chose system D_2; a design which achieves a slightly lower S/N ratio, but has a much larger potential to be further optimized in the tolerance-design stage. Hence, the proposed approach is valuable only if the S/N value of 16.76 for system D_3 is not satisfactory, and there is a need to proceed to the tolerance design stage. Note that the empirical models enable us to address the conflict imposed by the alternating values of the two decision criteria, namely, the SN ratio, and the entropy of the original systems. Another possibility to deal with such conflicts, which is left for future research, is to use one of the multi-criteria decision making methods [22]. Finally, note that the entropy values & bound of the naturalized systems reflect the *new spread* of factorial effects on the response variability, once the most influential factor has been naturalized. Thus, these H values are not necessarily lower than the H values of the nonnaturalized systems.

Let us now proceed to a short discussion on the potential use of the Huffman coding measure.

III. Huffman Coding: Naturalizing a Subset of Noise Factors

Once we adopt a "probabilistic approach" for the contribution of various noise factors to the variability in the system response, a series of probabilistic algorithms can be implemented into the robust design framework. In this section, we consider the Huffman coding algorithm for a specific related problem which is presented next.

Suppose that at some moment a severe disturbance in the system response occurs due to *one* of the noise factors that are represented in the outer array. The designer aims to discover, with as few experiments as possible, the disturbing noise factor in order to naturalize it. With no initial information, he follows the probabilistic approach suggested above, and considers the percent contribution of each noise factor as an estimate for the probability of that factor being the disturbing one. This is a reasonable assumption if no side information is available because the probability is spread among the noise factors proportionally to their affects on the response. The remaining question is how to organize the experiments to quickly reveal which factor is the disturbing one. If only a single factor can be naturalized in each experiment, a reasonable order of the experiments would be to start with the most probable factor, and proceed to less probable ones, as long as the disturbing factor is not found. For example, consider a case with six noise factors ordered according to their respective probabilities, as indicated by their PC_x values: 0.3, 0.2, 0.2, 0.2, 0.05, and 0.05. Then, the *expected number of experiments* under such ordering of the experiments would be $0.3\times 1+0.2\times 2+0.2\times 3+0.2\times 4+0.1\times 5 = 2.6$ experiments. That is, having a probability of 0.3 to discover that the disturbing factor is factor no. 1 in the first experiment; a probability of 0.2 to discover that the disturbing factor is factor no. 2 in the second experiment, etc., up to a probability of 0.1 to discover in the fifth experiment which of the factors, 5 or 6, is the disturbing factor (the sixth experiment is superfluous due to elimination). Consider now a realistic situation where *several* noise factors can be jointly naturalized during the experiments. The question remains, namely, how to organize those "group tests", as they are called in the literature [23]–[25], in order to minimize the expected number of test rounds. One can, for example, specify a testing procedure where approximately half of the factors are naturalized in each experiment (rounding up the number of the first subset, if it is odd). Such a procedure can be well described by a binary search tree [23]–[25]. The tree is a graphic representation of successive divisions of a set of items (noise factors in this case) into two subsets after each test/experiment. Fig. 1 presents, for example, a binary search tree with the above-considered six noise factors, labeled at the terminal nodes (leaves) by their numbers. The probability of noise factors x to be the disturbing factor is estimated by their PC_x values (using (4)), given at the bottom of the leaves within parentheses. In each test, a subset of noise factors are naturalized; then if the disturbance in the response disappears, it is known that the disturbing factor belongs to this subset. If the disturbance does not disappear, it is known (by elimination) that the disturbing noise factor belongs to the other (nonnaturalized) subset. The arcs in the tree represent the possible sequences of tests to identify the disturbing factor. Based on the outcome of the tests, the relevant subset of noise factors (that contains the disturbing factor)

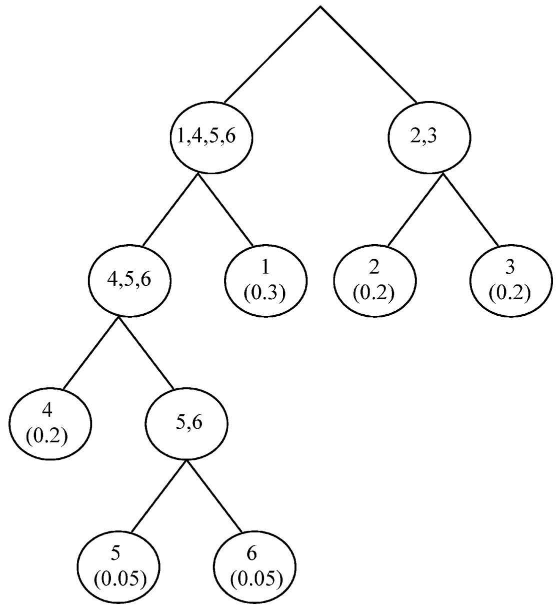

Fig. 2. The optimal binary test tree as obtained from the Huffman algorithm with an average number of $L = 2.4$ tests.

is further partitioned in the next experiment into two new subsets, such that the first subset includes half of the factors (again, rounded up). As seen from Fig. 1, in the first experiment, the designer naturalizes either the subset of noise factors $\{1, 2, 3\}$, or the subset of noise factors $\{4, 5, 6\}$. Then, for example, if the disturbed response belongs to the left subset, a second test is performed by naturalizing either noise factors $\{1, 2\}$, or noise factor $\{3\}$. If the result of the second test indicates that the disturbed response is noise factor $\{3\}$, an event with probability 0.2, then the disturbing factor was found after two rounds of tests. However, if the result of the second test indicates that the disturbed factor belongs to the subset $\{1, 2\}$, a third test has to be performed to indicate which of them is the noisy factor. The expected number of rounds of tests, L, is equal in this example to $\mathrm{L} = 0.3\times3+0.2\times3+0.2\times2+0.2\times3+0.05\times3+0.05\times2 = 2.75$ tests.

Note that the tree represents a testing strategy which is far from being optimal; it even obtains a worse result than the "single factor at a time" considered above. It is evident that an efficient testing policy should assign shorter testing branches to less reliable factors, and vice versa because this will result in a lower expected number of tests. However, in this example, it is seen that the suggested strategy fails to do so. For example, if factor no. 1, with the highest probability (0.3) of being the disturbing factor, is indeed the one, it will be discovered after three tests; whereas if factor number 6, with the lowest probability (0.05) of being the disturbing factor, is the one, it will be discovered after two tests.

The entropy measure of the PC_x values represents the lower bound on the expected number of tests [20], [21], and equals in this case to 2.35 tests. However, the entropy lower bound is unattainable in this case, and the optimal testing procedure is given by the well known Huffman coding algorithm [26]. The constructible Huffman algorithm is presented in Appendix II for purpose of self containment. In this example, it obtains an expected number of $L = 0.3 \times 2 + 0.2 \times 2 + 0.2 \times 2 + 0.2 \times 3 + 0.05 \times 4 + 0.05 \times 4 = 2.4$ tests, as shown in Fig. 2. The example given in Table I is too small for our pedagogical purpose. Yet, for example, the use of the Huffman algorithm on the $\mathrm{PC_x}$ values of system D_3 (neglecting the error term) implies that Factor B should be investigated in the first experiment. If factor 3 is not the disturbing factor, then the second experiment should be focused on factor A or C. Such procedure results in $L = 0.494 \times 1 + (0.361 + 0.128) \times 2 = 1.472$ tests.

In summary, it is seen that in a framework of probabilistic sequential experiments, one potential direction is to apply the Huffman coding to the PC_x measures in order to determine which of the noise factors should be naturalized. The Huffman algorithm which assigns long codes to less probable symbols, can be applied in the context of robust design to assign short testing sequences to noisier factors. Further details & ideas regarding the analogy between experiment procedures & coding methods can be found in [25], [27].

Appendix I

Let us assume that the number of noise factors q can be partitioned into two disjoint subsets containing exactly $0.8q$, and $0.2q$ factors. Then, according to the Pareto principle, good candidates for the *tolerance design* stage are those systems where 20% of the noise factors contribute 80% of the variability. Assuming that within each subset the factors' effects are identical, we can compute their respective percentage from the entropy upper-bound given in (6)

$$\begin{aligned} g(q) &= -\sum_{i=1}^{0.8q} \frac{0.2}{0.8q} \log_q \frac{0.2}{0.8q} - \sum_{i=1}^{0.2q} \frac{0.8}{0.2q} \log_q \frac{0.8}{0.2q} \\ &= -\,0.2 \log_q \frac{1-d}{0.8q} - d \log_q \frac{d}{0.2q} \\ &= h_q(0.8) + 0.2 \log_q 0.8q + 0.8 \log_q 0.2q \end{aligned} \tag{9}$$

where $h_q(x)$ is the binary entropy to the base $q : h_q(x) = -x \log_q x - (1-x) \log_q (1-x)$. Given the number of factors q, the designer can look for any system with an entropy percentage given by (6), which is less than or equal to $g(q)$.

With regard to the example in Table I, note that the entropy bound for systems with 4 noise sources (3 noise factors & an error term) is 40%.

Appendix II

An optimal code, defined by shortest expected length, for a given discrete distribution of items can be constructed by a simple algorithm discovered by Huffman [26]. The code can be directly mapped to a test tree, and thus, the algorithm is optimal for the considered search problem [21], [24]. The rationale of the algorithm is to assign long codes (test branches in the test tree) to less probable items (the noise factors in our case), and vice versa. The goal is to minimize the expected code (test) length, $L = \sum_i l_i \cdot p_i$, where l_i is the length of the branch to the ith leaf that represents the ith item, and p_i is the probability of this item to be the searched item. The proof of optimality can be found in [26] or [21], and is omitted here.

The Huffman algorithm for optimal codes is presented next.

TABLE V
THE HUFFMAN CODING ALGORITHM

X_i	P^1	P^2	P^3	P^4	P^5	Code iter.1	Code iter.2	Code iter.3	Code Iter.4	Code (final)
x_1	0.3	0.3	0.3	0.4	0.6				1	11
x_2	0.2	0.2	0.3	0.3	0.4			1	1	01
x_3	0.2	0.2	0.2	0.3				0	0	00
x_4	0.2	0.2	0.2				1	1	01	101
x_5	.05	0.1				1	01	01	001	1001
x_6	.05					0	00	00	000	1000

The Huffman Coding Procedure

For a given set of items (noise factors in our case) $\mathrm{X} = \{x_i\}$ where $|\mathrm{X}| = N$, with a known distribution $p(x_i) = p_i$, and a binary coding alphabet {0,1}, the coding algorithm is the following one:

1. Arrange all source symbols in decreasing order of probabilities: $p_1 \geq p_2 \geq \ldots p_N$.
2. Assign "0" to the last digit of the last code-word, denoted as C_N, and "1" to the last digit of the previous code-word C_{N-1}.
3. Add p_N to p_{N-1} to obtain a new set of probabilities $p_1, p_2, \ldots, p_{N-2}, p_{N-1} + p_N$.
4. Repeat all the steps for this new set of probabilities.

Example: Table V illustrates the Huffman algorithm for binary coding. The algorithm steps are given in columns 2–6, and their respective codes are given in columns 7–11. The final code in column 11 can be directly mapped to a binary test tree. The code is read from left to right. In each test stage, the set of items is partitioned into two subsets in correspondence to the "1" & "0" symbols in the code. In this example, the first test should partition the whole set of items (factors in our case) into two subsets: $\{x_1, x_4, x_5, x_6\}$ having "1" in the most left digit of their codes, and $\{x_2, x_3\}$ having "0" in the most left digit of their codes. If the searched item (disturbing factor) belongs to the first subset, the next test divides this subset to two new subsets according to the second digit in their codes, thus into $\{x_1\}$ & $\{x_4, x_5, x_6\}$. The resulting binary test tree is presented in Fig. 2, and obtains an expected number of $L = \sum_i l_i \cdot w_i = 2 \cdot (0.3 + 0.2 + 0.2) + 3 \cdot 0.2 + 4 \cdot (0.05 + 0.05) = 2.4$ tests.

REFERENCES

[1] G. Taguchi, "Off-line and on-line quality control systems," in *Proc. Int. Conf. Quality*, Tokyo, Japan, 1978.

[2] D. M. Byrne and S. Taguchi, "The Taguchi approach to parameter design," *Quality Progress*, vol. 20, no. 12, pp. 19–26, 1987.

[3] M. S. Phadke, *Quality Engineering Using Robust Design*. Englewood Cliffs, New Jersey: Prentice Hall, 1989.

[4] G. Box, "Signal-to-noise ratios, performance criteria and transformation," *Technometrics*, vol. 30, no. 1, pp. 15–40, 1988.

[5] R. V. Leon, A. C. Shoemaker, and R. N. Kacker, "Performance measures independent of adjustment," *Technometrics*, vol. 29, no. 1, pp. 253–285, 1987.

[6] D. M. Steinberg and D. Bursztyn, "Dispersion effects in robust-design experiments with noise factors," *J. Qual. Technol.*, vol. 26, no. 1, pp. 12–20, 1994.

[7] P. J. Ross, *Taguchi Techniques for Quality Engineering*. New York: McGraw-Hill, 1988.

[8] A. Rupe and S. Gavirneni, "Designing for high performance of a command, control, communication and information network," *IEEE Trans. Reliab.*, vol. 44, pp. 230–236, Jun 1995.

[9] G. Taguchi, "Quality engineering (Taguchi methods) for the development of electronic circuit technology," *IEEE Trans. Reliab.*, vol. 44, pp. 225–229, Jun 1995.

[10] G. Box and R. D. Meyer, "Dispersion effects from fractional designs," *Technometrics*, vol. 28, no. 1, pp. 19–27, 1986.

[11] G. Box, "Signal-to-noise ratios, performance criteria and transformation," *Technometrics*, vol. 30, no. 1, pp. 1–17, 1988.

[12] J. Pignatiello, "Strategies for robust multiresponse quality engineering," *IIE Trans.*, vol. 25, no. 3, pp. 5–15, 1993.

[13] D. M. Steinberg, "Robust design: experiments for improving quality," in *Handbook of Statistics*, S. Ghosh and C. R. Rao, Eds: Elsevier Science, 1996, vol. 13, pp. 199–240.

[14] S. D. McCaskey and K. L. Tsui, "Analysis of dynamic robust design experiments," *Int. J. Production Res.*, vol. 35, no. 6, pp. 1561–1574, 1997.

[15] R. S. Kenett and S. Zacks, *Modern Industrial Statistics: Design and Control of Quality and Reliability*: Brooks/Cole Publishing Company, 1998.

[16] K. L. Tsui, "Modeling and analysis of dynamic robust design experiments," *IIE Trans.*, vol. 31, pp. 1113–1122, 1999.

[17] S. M. Sanchez, "Robust design: seeking the best of all possible worlds," in *Proc. 2000 Winter Simulation Conf.*, 2000, pp. 69–76.

[18] J. S. Hunter, "Statistical design applied to product design," *J. Qual. Technol.*, vol. 17, no. 4, pp. 210–221, 1985.

[19] M. J. Anderson and S. L. Kraber, Using Design of Experiments to Make Processes More Robust to Environmental and Input Variations, Paint and Coating Industry (PCI), pp. 1–7, 2002.

[20] C. E. Shannon, "The mathematical theory of communication," *Bell System Technical Journal*, vol. 27, pp. 379–423, 1948. and pp. 623–656.

[21] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York: Wiley & Sons, 1995.

[22] E. Triantaphyllou, *Multi-Criteria Decision Making Methods: A Comparative Study*: Kluwer Academic Publishers, 2000.

[23] D. Z. Du and F. K. Hwang, *Combinatorial Group Testing and Its Applications*, Singapore: World Scientific, 1993.

[24] I. Ben-Gal, "An upper bound of the weight-balanced testing procedure with multiple testers," *IIE Trans. Qual. Reliab.*, vol. 36, no. 5, pp. 481–493, 2004.

[25] R. Ahlswede and I. Wegner, *Search Problems*. New York: Wiley & Sons, 1987.

[26] D. A. Huffman, "A method for the construction of minimum redundancy codes," in *Proc. Institute of Radio Engineering*, vol. 40, 1952, pp. 1098–1101.

[27] I. Ben-Gal, Y. Herer, and T. Raz, "Self-correcting inspection sequences under inspection errors," *IIE Trans. Qual. Reliab.*, vol. 34, no. 6, pp. 529–540, 2002.

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