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Probabilistic sequential methodology for designing a factorial system with multiple responses

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This paper addresses the problem of optimizing a factorial system with multiple responses. A heuristic termed *probabilistic sequential methodology* (PSM) is proposed. The PSM identifies those designs that maximize the likelihood of satisfying a given set of functional requirements. It is based on sequential experimentation, statistical inference and a probabilistic local search. The PSM comprises three main steps: (1) screening and estimating the main location and dispersion effects by applying fractional factorial experiments (FFE) techniques; (2) based on these effects, establishing probabilistic measures for different combinations of factor-levels; and (3) constructing a set of candidate designs from which the best solution is selected by applying a heuristic local search. The PSM is attractive when the exact analytic relationship between factor-level combinations and the system's responses is unknown; when the system involves qualitative factors; and when the number of experiments is limited. The PSM is illustrated by a detailed case study of a Flexible Manufacturing Cell (FMC) design.

1. Introduction

1.1. *Sequential experimentation strategies for systems with multiple responses*

Consider the problem of optimizing a factorial system in situations where the exact analytic relationship between the system configuration (characterized by a combination of factor-levels) and the system response is unknown. Consequently, factorial effects are evaluated by experimentation. One practicable strategy is to conduct all experiments at once, while the other approach is to run experiments sequentially. Response Surface Methodology (RSM), for example, is founded on such an approach (e.g. see Myers and Montgomery 1995).

Box (1992) mentions several strategies by which a second stage of experimentation might evolve as a result of the analysis of the first stage. For 'scientific understanding', Box suggests adding a second (full or fractional) factorial experiment. For a 'quick fixing' of factors, he proposes a 'picking-the-winner' strategy, where the additional experiments do not necessarily form a fractional factorial experiment. Box analysis, which relates to a specific example, addresses only location effects (effects on the response mean), overlooking the dispersion effects (effects on the response variance, which are addressed by Box and Meyer (1986a) but not in the context of sequential experimentation).

Driven by similar concepts, Shang (1995) suggests sequentially employing 'two optimum-seeking methods' for designing and optimizing a flexible manufacturing

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system: first, the controversial Taguchi method (e.g. see Box 1988, Box *et al.* 1988, Nair 1992) is applied as a screening process and used to optimize qualitative factors. Second, the RSM is applied in order to fine-tune the quantitative and continuous factors, since RSM 'cannot perform optimization for qualitative factors'. Shang's use of the RSM approach requires a large number of experiments. In the case study provided, approximately 200 simulation-runs are performed to propose the best design with respect to a single performance measure. Such an extensive experimentation is suitable only when the cost per experiment is sufficiently low.

Both Shang and Box address, in the above mentioned papers, systems with a single response. However, many systems have more than one criterion by which their overall performance is determined. Extensive research work was conducted in this area. Pignatiello (1993), for example, defines a quadratic loss function for use of multiple quality characteristics and summarizes several strategies that could be employed when seeking robust designs. Elsayed and Chen (1993) as well as Logothetis and Haigh (1988) used Taguchi's loss function and Box's PerMIA to optimize a multi-response system. Other methods suggest converting all objectives into equivalent units by considering their relative importance.

The approach presented in this paper proposes a numerical probabilistic measure termed the *success probability* (see Suh 1990, 1995, Braha and Maimon 1998). This measure quantifies the likelihood of a design to satisfy a set of independent requirements. The use of the *success probability* measure provides a means by which different functional requirements are integrated and normalized to units of probability. Using this measure for design optimization is appealing for several reasons: (1) it offers an intuitive and unbiased decision criterion to support the design process (as opposed to coded units used frequently by the RSM and other methods); (2) it is applicable to systems composed of both quantitative and qualitative (non-ordinal) factors; and (3) the use of the probabilistic measure enables the development of many probabilistic algorithms (see section 3) including the heuristic optimization proposed by this paper.

1.2. *The proposed heuristic*

In this paper, we suggest a heuristic termed *Probabilistic Sequential Methodology* (PSM) for a successful factorial design, which is based on sequential experimentation, statistical inference and probabilistic local search. The PSM applies fractional factorial experiments (FFE) techniques combined with a heuristic local search. The FFE method is used in the first stage for factor screening, and for estimating the main location's as well as the dispersion's effects. Based on these effects, the PSM establishes the success probability measures, which evaluate the likelihood of each design (or sub-design) to satisfy the requirements. Then, a heuristic local search is applied and a sample set of 'most probable designs' is constructed and evaluated. The PSM addresses multiple requirements by repeating the local search with respect to different functional requirements. Finally, the PSM finds the best design, with respect to all the functional requirements, among the sets of 'most probable designs'.

The PSM is formulated in a generic manner, and although further research is needed to support the hypothesis that the heuristic is applicable for general problems, a successful application is presented in section 4. The rational of the proposed heuristic is based on the following fundamentals: (1) it utilizes a sequential experimentation paradigm; (2) it considers both dispersion as well as location effects; (3) it applies, similar to Shang's procedure, both a screening stage (using FFE instead of the controversial Taguchi method) as well as a fine tuning stage (using a similar strategy to Box's 'picking-the winner' approach); (4) its optimization heuristic is applicable to systems having numerous qualitative and quantitative factors; and (5) it addresses systems that involve multiple performance measures. Note that the PSM generates (heuristically) the 'set of most probable design' (the final *region of interest*) after a *single* experiment, as opposed to the iterative *steepest ascent* methodology used in RSM (particularly in a multiple responses case when a small step size is applied). However, in contrast to RSM, if a design point that is included in the 'set of most probable designs' falls far from the region containing the original observations, then the prediction interval associated with its response might be incorrect and substantially large. This fact motivates the use of replicated confirmation experiments for each design in that set, as done in the final steps of the PSM.

It follows that the PSM is attractive for certain situations. First, when the designer is willing to sacrifice knowledge about the prediction properties of the model in the final region of interest. Secondly, when experiments are too expensive for an iterative optimization process while replications are relatively cheap (e.g. in testing of design prototypes, where the construction of a prototype is costly while the cost of its testing is relatively low). Thirdly, when the design contains many qualitative factors. For models that mainly involve continuous quantitative factors, more suitable optimization methods may be applied, such as (1) multiple RSM models (one per each level of a qualitative factor) combined with a direct optimization by steepest ascent procedures of a properly chosen quadratic loss function (Pignatiello 1993); or (2) the *dual response* surface approach (Myers and Montgomery 1995) by modelling and optimizing both the mean and the variance of a response.

The paper is organized as follows: notations and problem formulation are provided in section 2, followed by a detailed discussion of the proposed methodology in section 3. The solution approach is illustrated in section 4 by designing a real world Flexible Manufacturing Cell (FMC). Section 5 concludes the paper.

2. Notation and problem formulation

Consider an empirical model written as

$$y = f(x_1, x_2, \dots, x_K; \hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_p) + \varepsilon, \tag{1}$$

where y is the system response; f is the first- or second-order polynomial; $x_1, \dots x_K$ are the system factors; ε is a random noise component; and $\hat{\beta}_1, \dots, \hat{\beta}_p$ are the parameter estimators. In particular, if $x_1, \dots, x_K$ are qualitative or nominal quantitative factors, then a design is determined by a combination of parameter-levels:

$$d_n \equiv \{x_1(q_1), x_2(q_2), \dots, x_K(q_K) | q_i \in \{1, \dots, Q\}\} \quad n = 1, \dots, Q^K, \tag{2}$$

where each factor takes one out of Q possible levels. For $Q = 2$, we simply use $x_i(+)$ or $x_i(-)$ to denote the levels of the ith factor. Consider a multiple responses system with L responses and W replications per experiment. Denote the response of the wth replication of the nth treatment, with respect to the lth functional requirement by y^l_{nw}, $l = 1, \dots, L$, $w = 1, \dots, W$ (the notations w, l are omitted if $W = 1$ or $L = 1$, respectively). Assume that a functional requirement is represented by a required tolerance for the respective response. Let $t_l = (LB_l, UB_l)$ be the lth required tolerance, where LB_l and UB_l denote the lower and upper limits, respectively. $T = \{t_l | l = 1, 2, \dots, L\}$ denotes the set of required tolerances in the L multiple-responses system.

As mentioned above, we consider situations in which the designer looks for the combinations of levels that maximize the likelihood of satisfying the functional requirements. This likelihood is evaluated by the *success probability* (Suh 1990), which is the probability of the design response to fall within the required tolerance. P_n^l denotes the *success probability* of the nth design with respect to the lth functional requirement, and is given by

$$p_n^l \equiv \text{Prob}\left[LB_l \le y_n^l \le UB_l\right]. \tag{3}$$

In practice, the exact probability distribution of the response might be unknown and has to be estimated empirically. In which case, the estimated success probability $\hat{p}_n^l$ is found by substituting y_n^l with the estimated response $\hat{y}_n^l$. The *overall success probability*, P_n, of the nth design with respect to a set of L independent functional requirements is estimated by

$$\hat{P}_n = \prod_{l=1}^{L} \hat{P}_n^l. \tag{4}$$

Finally, let us address scenarios where experimental resources are limited. We associate a unit cost to a single experiment run, and limit the total number of experiments to $\mathbb{C}$. The designer task is to seek the 'best' combination of factor-levels, constituting the 'best' design solution d^*, which maximizes the *overall success probability* by performing at most $\mathbb{C}$ experiments. Thus, $d^* \equiv d_{n^*}$, where $n^* \equiv \arg\max_n\{\hat{P}_n | n = 1, \ldots, Q^K\}$. Note that by 'best' we mean 'the best solution considering the information gathered throughout the experiments'. The designer may decide to examine the admissibility of the optimal design d^* by comparing its *overall success probability* p^* with a threshold probability P_0. If $P^* < P_0$, an elaborate experimentation of the system, with a larger number of factors and/or levels might be required.

3. A probabilistic sequential methodology for design

The search for an optimal design may be deterministic, by showing which designs are categorically inferior, or probabilistic, by identifying those designs that have the highest probability of satisfying a given set of requirements (see Maimon and Braha 1996). The methodology presented in this paper is founded on the probabilistic paradigm. This paradigm enables the development of many other probabilistic schemes, such as: (1) using *success probability* measures to evaluate designs according to their functional complexity (Suh 1990, 1995); (2) introducing a stochastic dynamic programming framework to design experiments (Ben-Gal and Caramanis 1998); and (3) developing optimal information criteria that are based on the correspondence between FFE and Information Theory (Ben-Gal and Levitin 1997, 1998).

3.1. *Estimation of success probability measures*

In this section, we discuss the procedure used to estimate the *success probability* of different combinations of factor-levels with respect to a single functional requirement. Later, this procedure is repeated for each independent requirement and the *overall success probability* is computed by applying (4).

The *success probability* of the nth design depends on the probability distribution of its response, y_n, which is estimated empirically. It is assumed that the response is normally distributed, therefore, it is enough to estimate its mean and variance in order to evaluate $\hat{p}_n$. If the normality assumption does not hold, one has to increase the number of replications and redefine the success probability with respect to the mean response $\bar{y}_n$, which is Gaussian in distribution according to the central limit theorem. In what follows, two alternative procedures are applied in order to estimate the response mean and the response variance.

Direct estimation: If the experiments have a sufficient number of replications (say, $W \geq 5$), the response statistics of the nth treatment in the design matrix can be directly estimated by the sample mean and sample variance, given respectively by

$$\bar{y}_n = \frac{1}{W}\sum_{w=1}^{W} y_{nw} \tag{5}$$

and

$$S_n^2 = \frac{1}{W-1}\sum_{w=1}^{W}(y_{nw} - \bar{y}_n)^2. \tag{6}$$

Indirect estimation: Alternatively, one might use the empirical model to estimate the response statistics. Consider, for example, the first-order *linear statistical model* with two levels:

$$y = \beta_0 + \sum_{i=1}^{K} \beta_i x_i(q_i) + \sum_{i=1}^{K-1}\sum_{j=i+1}^{K} \beta_{ij} \cdot x_i(q_i) x_j(q_j) + \varepsilon; \quad q_i, q_j \in \{+, -\}; \quad \varepsilon \sim N(0, \sigma^2), \tag{7}$$

where model parameters $\beta_1, \ldots, \beta_p$ are estimated by applying regression techniques (e.g. see Montgomery 1997). Then, the expected response for *any* factor combination (whether or not such a combination is included in the design matrix) can be estimated by adding the location effects associated with this particular combination of parameter levels. Clearly, this estimate depends on the model adequacy that has to be verified. The method of estimating the response variance depends on the existence of dispersion effects. If no dispersion effects are found (e.g. as implied by equation (7)), the response variance σ^2 is simply estimated by the mean square error (MSE). If dispersion effects do exist, the variance response is estimated by the sum of dispersion effects associated with a particular combination of factor-levels. Box and Meyer (1986a) provide an approximate method to estimate the dispersion effects that is based on calculating the sample variance of different subsets of residuals. Each subset of residuals is associated with a specific factor-level setting. For instance, consider a 2^{K-G} FFE, where G is the number of generators. A significant difference between the variance of the 2^{K-G-1} residuals $S^2[x_i(-)]$, where the factor x_i is fixed to its low level, and the variance of the remaining 2^{K-G-1} residuals $S^2[x_i(+)]$, where the factor x_i is fixed to its high level, indicates that the factor x_i has a dispersion effect. x_i is then called a *dispersion factor*. In practice, dispersion effects are identified by plotting and examining the residuals against the levels of a particular factor, or by applying the statistic

$$F^*_{x_i} = \ln \frac{S^2[x_i(+)]}{S^2[x_i(-)]}, \tag{8}$$

which has an approximate normal distribution if the two variances $\sigma^2[x_i(+)]$ and $\sigma^2[x_i(-)]$ are equal. $F^*_{x_i}$ values may be studied by numerical comparisons and by normal probability plots.

In the PSM, we extend Box and Meyer's method to account for the combination of factor-levels. The approximate method is applied to estimate dispersion effects of factor interactions. Note, however, that in order to estimate the sample variance of residuals, one needs enough data points for each combination of levels. In general, for a 2^{K-G} FFE with W replications and k_d interacting dispersion factors, there are $W \times 2^{K-G-k_d}$ data points for each subset of residuals (associated with a combination of k_d factors). The number of data points for each subset of residuals decreases as the number of interacting factors increases. Thus, when estimating higher-order dispersion effects directly, one expects a lower accuracy of the variance estimates. Moreover, the number of required calculations grows exponentially with the order of the interaction. In particular, as illustrated in section 4.3, approximating the dispersion effects of k_d factors (each with Q levels) up to the δth-order, requires estimating the variance of

$$\binom{k_d}{\delta} Q^{\delta}$$

subsets of residuals.

Thus far we have discussed the *success probability* of designs as determined by an entire set of factor combinations, $d_n \equiv \{x_i(q_i) | i = 1, \ldots, K;\ q_i \in \{1, \ldots, Q\}\}$. We now consider sub-designs represented by a subset of factor combinations:

$$d[x_i(q_i), \ldots, x_j(q_j)] \equiv \{x_i(q_i), \ldots, x_j(q_j) | i, \ldots, j \le K;\ i \ne \ldots \ne j;\ q_i, \ldots, q_j \in \{1, \ldots, Q\}\}. \tag{9}$$

Here, the expected response $E[\hat{y}(x_i(q_i), \ldots, x_j(q_j))]$ and the response variance $V[\hat{y}(x_i(q_i), \ldots, x_j(q_j))]$ can be estimated, respectively, by the sum of location effects and the sum of dispersion effects. The other parameters that do not belong to the subset are set to zero. This procedure results in an expected response of all the designs that contain such a subset (while all other factors vary uniformly among all possible combinations).

Finally, by knowing both the location and the dispersion effects of a sub-design, one can estimate its *success probability* as follows,

$$\hat{\mathrm{P}}[x_i(q_i), \ldots, x_j(q_j)] \equiv \mathrm{Prob}\{\hat{y}[x_i(q_i), \ldots, x_j(q_j)] \in t\}. \tag{10}$$

In particular, the *success probability* measures of a subset that contain a single factor with two levels are estimated accordingly and denoted by $P[x_i(+)]$, $P[x_i(-)]$.

3.2. *The PSM algorithm steps*

Having defined the *success probability* measure and delineated ways to estimate it, we outline the PSM algorithm. The algorithm is further illustrated in section 4 by presenting a detailed case study.

Outline of the PSM algorithm

Step 1. Initialization

Consider a factorial system, L independent functional requirements (provided by the tolerances set T) and a limit $\mathbb{C}$ on the total number of experiments. Allocate N treatments out of $\mathbb{C}$ permitted experiments, and set an initial design matrix $\mathbf{X}$ that contains K factors, which are assumed to affect the first functional requirement (i.e. set $l = 1$). Different FFE criteria (e.g. alphabetic optimality criteria) can be applied to generate various design matrices. Nonetheless, the PSM requires main factors and interactions not to be aliased one with the other, so their effects can be estimated independently. Noise factors might be included in the design matrix, if such factors exist, to account for robust designs in a later stage.

Step 2. First experimentation (screening)
Conduct the experiments in $\mathbf{X}$ by utilizing either the original system or a model of it (e.g. computer simulation).

Step 3. Analysis

(i) Identify the significant factors and interactions having location effects using normal probability plots or other FFE techniques (see Box and Meyer 1986b). These factors are called *location factors* and are represented by the vector $\mathbf{x}_a$ (following Pignatiello 1993).

(ii) Identify significant factors and interactions having dispersion effects using the sample variance of residuals, plots of residuals versus factors, or other methods. These factors are called *dispersion factors* and are represented by the vector $\mathbf{x}_d$.

(iii) Identify factors having no location effects and no dispersion effects. These factors are called *non-significant factors* and are represented by the vector $\mathbf{x}_0$.

(iv) Verify the underlying statistical model by analysing the residuals and variances (ANOVA). If needed, refine the statistical model, conduct more experiments, and repeat Steps 1–3. If the model is validated, estimate the parameters of the underlying model.

Step 4. Fixing location and non-significant factors
Fix the factors and interactions having *only* location effects $(x_i, x_j, x_i x_j, \ldots \in \mathbf{x}_a, \notin \mathbf{x}_d)$ to their best levels, with respect to functional requirement t_l. Fix the factors and interactions that have *both* location and dispersion effects $\left(x_i, x_j, x_i x_j, \ldots \in \mathbf{x}_a \cap \mathbf{x}_d\right)$ to the levels that will maximize the estimated *success probability* (applying equation (10)), with respect to the functional requirement t_l. Fix the factors and interactions having no location effects and no dispersion effects $\left(\forall x_i \in \mathbf{x}_0\right)$ to their best economic levels.

Step 5. Designing the second set of experiments
Construct a new design matrix $\mathbf{X}'$ that includes N' new treatments as follows.

(i) If the dispersion factors appear to interact among themselves, conduct a full factorial experiment, provided that N' is large enough, by varying all the dispersion factors $\left(x_i \in \mathbf{x}_d\right)$ while all other factors $\left(x_i \in \mathbf{x}_a \cup \mathbf{x}_0\right)$ are fixed according to Step 4.

(ii) If N' is *not* large enough, *or* if the dispersion factors, $x_i \in \mathbf{x}_d$, do not appear to interact one with the other, a set of N' designs that have the highest estimated *success probability* (the set of 'most probable designs') is generated heuristically. To compute the *success probability* measures two cases are considered.

(*a*) If there are sufficient data points to estimate the variance of subsets of residual, use the direct estimate of dispersion effects for each combination of levels of the dispersion factors, as suggested in section 3.2 (based on Box and Meyer 1986a).

(*b*) If data are not sufficient to support a direct evaluation of each level-combination of dispersion factors, approximate the dispersion effect of each combination by the sum of its individual dispersion effects (this method is illustrated in section 4.3).

Step 6. Second experimentation
Repeat each experiment in $\mathbf{X}'$ for W' independent replications. Then, estimate the *success probability* $\hat{P}_n^l$ associated with each design in $\mathbf{X}'$ as follows; if W' is large enough (i.e. $W' \geq 3$) compute $\hat{P}_n^l$ directly (applying equations (5) and (6)), otherwise use the underlying statistical model and estimate it indirectly (applying equation (10) as suggested in section 3.1).

Step 7. Repeating Steps 1–6 for different functional requirements
Generate the set $\mathbf{X}_l'$ of 'most probable designs' with respect to each functional requirement $l\ (l = 2, \ldots, L)$ by repeating Steps 1–5, and apply Step 6 to each set $\mathbf{X}_l'$.

Step 8. Selecting the best design
Considering that the L functional requirements are independent, obtain the mega-set $\mathbf{X}^L = \cup_{l=1}^{L} X_l'$ of design points by merging the L sets of 'most probable designs'. Exclude those design points in $\mathbf{X}^L$ that are expected to have a low overall success probability and obtain the filtered mega-set $\tilde{\mathbf{X}}^L$. Experiment and replicate each design point in the filtered mega-set $\tilde{\mathbf{X}}^L$ with respect to the remaining functional requirement. Compute the overall success probability of each design point in $\tilde{\mathbf{X}}^L$ by applying (5) and (6). Select the best design solution d^* that yields the maximum overall success probability among all the designs in $\tilde{\mathbf{X}}^L$.

The PSM algorithm is illustrated in figure 1.

4. An illustrative case study

In this section, a detailed case study of a Flexible Manufacturing Cell (FMC) design is presented. Designing a FMC is a complex task since the exact analytic relationship between the cell configuration and its performance (e.g. throughput rates) is unknown. For example, if a change is made at a particular workstation, the overall impact may not be predictable analytically. Consequently, experimentation through computer simulation is often used as a technique for evaluating the performance measures of FMCs (see Shang 1995). A typical FMC includes factors such as: number of machines, queue discipline, buffer sizes, operation modes, and machine groupings into cells. Common performance measures include throughput rates, work-in-process levels, machine utilization, and profitability.

The case study is organized as follows: (1) a brief description of the FMC, the underling production processes, and the factors are provided; (2) several issues related to the simulation modelling of the FMC are addressed; and (3) a detailed implementation of the PSM algorithm (described in section 3.2) is applied for the FMC design problem.

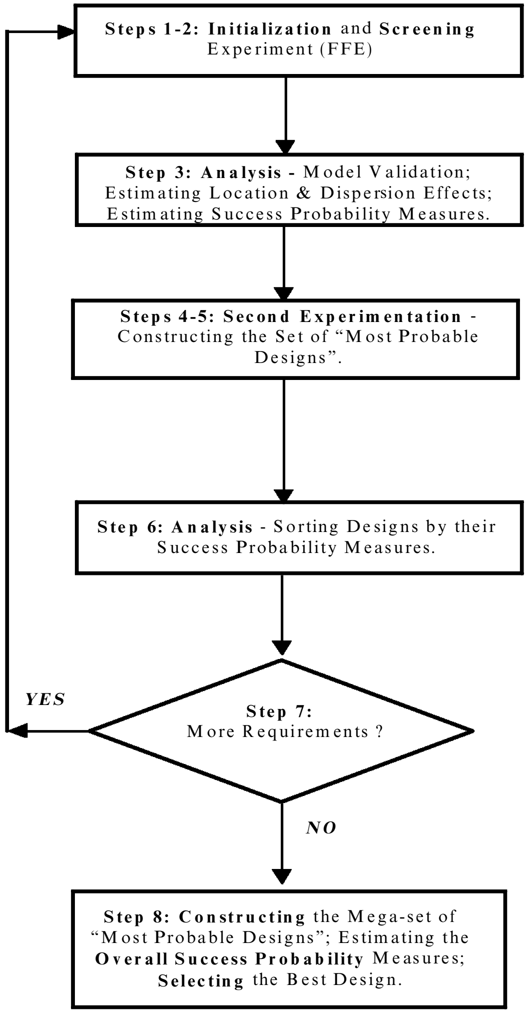

Figure 1. A schematic flowchart of the PSM algorithm.

4.1. *System description*

4.1.1. *Cell stations and machines*

The FMC is located at the Automated Design & Manufacturing Laboratory (ADMS) at Boston University. The cell is composed of various types of stations located around a central conveyor belt. It includes two robots; a lathe machine (DYNA); two milling machines (TMC); a vision system; and various control and communication components (such as sensors; terminals; communication networks; controllers; and a PLC). The cell is composed of the following stations (see figure 2): (1) computer-aided design stations; (2) a computer numerically controlled (CNC) machines station; (3) a quality control (QC) station, which is used to perform quality inspection and rework processes; (4) an assembly station, which is used to assemble finished products; and (5) a main controller station, which is used to supervise the various activities in the FMC.

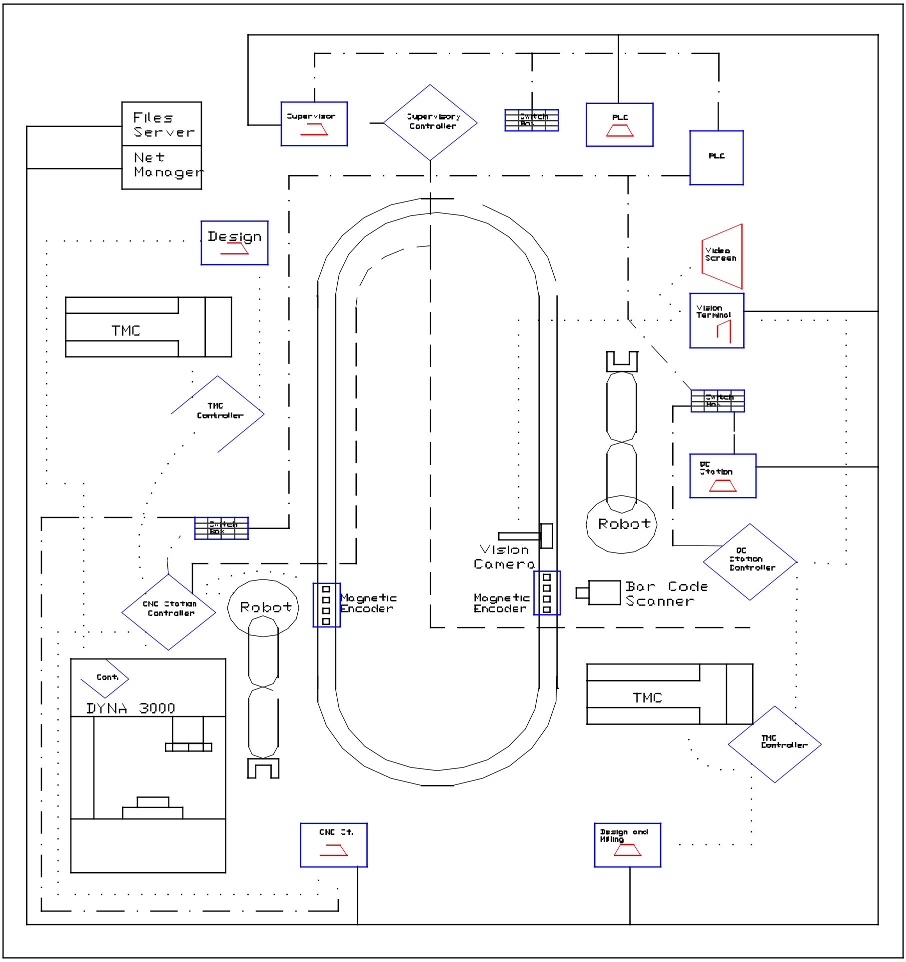

Figure 2. Schematic view of the Eshed Robotec flexible manufacturing cell.

Two different production processes are considered. In the first process, two different part types are manufactured and assembled to create a finished product. A third part type is manufactured in the second process. Each of the part types follows a different processing route as detailed below and illustrated in figure 3.

Part type 1 (process 1): Part arrival → loaded to TMC 1 by robot 1 → milling by TMC 1 → loaded to the conveyor by robot 1 → transportation by the conveyor → quality control by the vision system → re-milling in probability 0.3 by TMC 2 → assembly with part type 2 by robot 2.

Part type 2 (process 1): Part arrival → loaded to the DYNA by robot 1 → turning by the DYNA → loaded to the conveyor by robot 1 → transportation by the conveyor → quality control by the vision system → re-milling in probability 0.4 by TMC 2 → assembly with part type 1 by robot 2.

Part type 3 (process 2): Part arrival → turning by the DYNA → transportation by robot 1 → milling by TMC 1 → transportation by the conveyor → quality control by

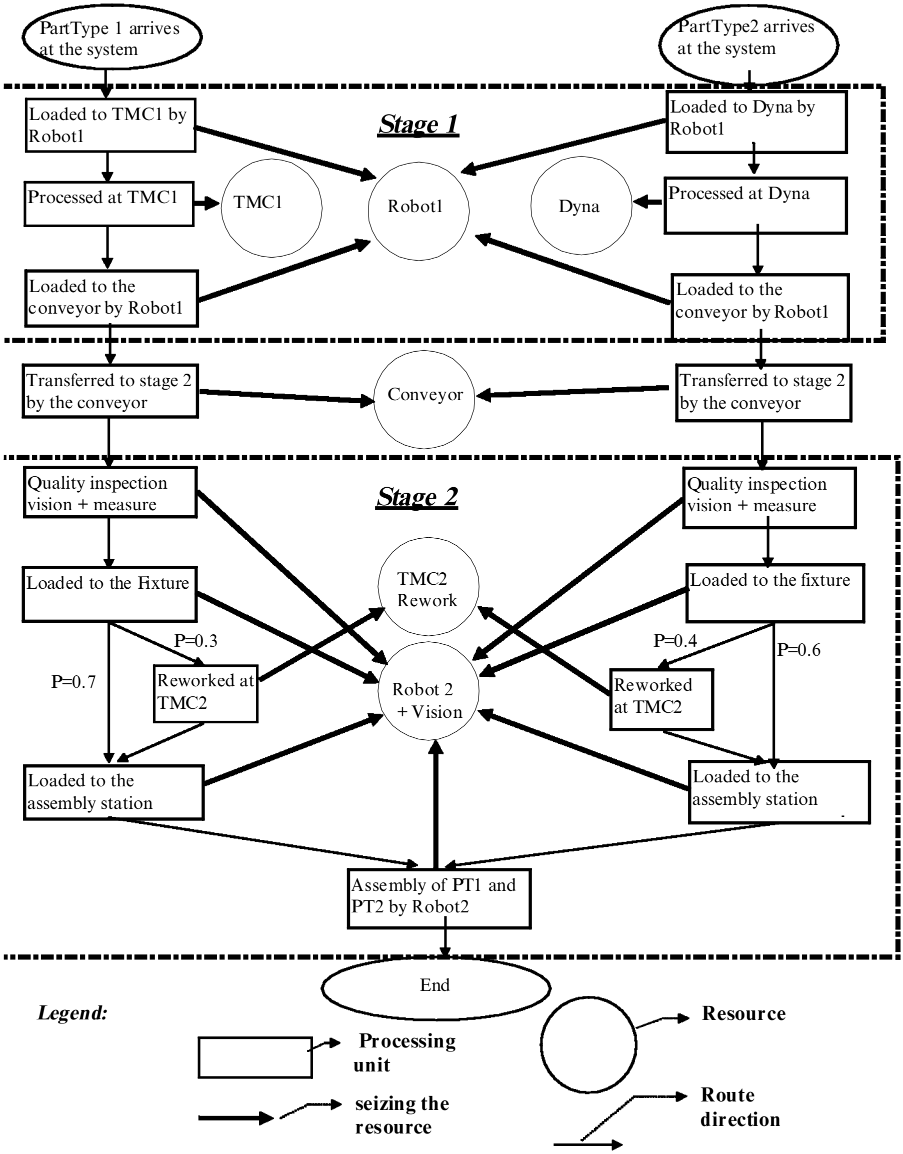

Figure 3. A schematic flowchart of Process 1.

the vision system → re-milling in probability 0.2 by TMC 2 → assembly of two identical part types by robot 2.

4.1.2. *The FMC control factors*

The following factors that affect the cell performance, are considered.

(1) Processing times of primary operations at the CNC station are fixed. Rework (countersinking and tapping) processing times are distributed exponentially but their mean differs on the raw material type (aluminium versus steel). For

aluminium parts, the mean is 5.5 time units, for steel parts, the mean is 6.5 time units.

(2) Two types of maintenance plans are considered for the robots. The first plan ensures a better reliability but requires more maintenance time. Reliability is measured in terms of *mean time between failures* (MTBF), and maintenance time is measured in terms of *mean time to repair* (MTTR). The reliabilities of the CNC machines are fixed.

(3) Two modes of operation of the central conveyor are considered. Mode 1 utilizes a belt speed of 10 m per time unit, and mode 2 utilizes a belt speed of 20 m per time unit.

(4) Robots perform certain material-handling tasks according to several scheduling procedures. Three policies are considered: (a) first-come first-serve (FCFS), (b) high value first (HVF)—prioritizing part type 2 over part type 1, and (c) low value first (LVF)—prioritizing part type 1 over part type 2. Note that, online operational control and offline design aspects are considered simultaneously, as suggested by Shang (1995) and Egbelu and Tanchoco (1984).

(5) Inter-arrival times of raw materials are normally (truncated) distributed with a mean arrival rate of 8 time units. The variances of the arrival rates depend on the supplier's service. Two possible scenarios are considered. If it is found that variance effects interact with a controllable factor, they might be treated as *noise* factors to establish a robust design according to the Taguchi method (it is suggested to include the noise factors in a combined array instead of a crossed structure array in order to minimize the number of experiments—e.g. see Phadke 1989). Otherwise, the variances are treated as controllable factors by supervising the supplier policies or by replacing them.

(6) The DYNA lathe machine can operate in two possible modes: (a) 'long stack' mode, which requires longer set-up time and longer time between consecutive set-ups (20 time units/150 times units); and (b) 'short stack' mode, which requires shorter set-up and shorter time between consecutive set-ups (8 time units/85 times units).

(7) Some second-order interactions are assumed to be significant.

Eight two-level factors are considered for further investigation, as summarized in table 1.

Factors	Code	Level 1 $(+)$	Level 2 $(-)$
1. Maintenance type of robot 1	R1	MTBF/MTTR (75/3)	MTBF/MTTR: 125/10
2. Maintenance type of robot 2	R2	MTBF/MTTR (75/3)	MTBF/MTTR: 125/10
3. Dyna modes	DY	'short Stack' (8/85)	'long stack' (20/150)
4. Conveyor modes	CS	MODE 1 (10)	MODE 2 (20)
5. Policy of robot 1	PR1	FCFS	LVF
6. Policy of robot 2	PR2	FCFS	HVF
7. Arrival rate variance	ARR	$\sim N(8, 1)$	$\sim N(8, 2)$
8. Raw material type	RM	Aluminium	steel

Table 1. The controllable factors used for the FMC design.

4.1.3. *The FMC performance measures*

Three performance measures are outlined as follows.

(1) *Average Flow Time for Process* 1 (*Rsys1*), which is the average time that a product spends in a manufacturing system during process 1. *Rsys1* is a smaller-the-better (STB) type of measure. *Rsys1* also measures the balance between production rates of part type 1 and part type 2, since only the assembled finished products are released from the system.
(2) *Average work-in-process in process* 1 (*WIP1*), which is the amount of inventory (parts) in the manufacturing system during process 1. *WIP1* is a nominal-the-best (NTB) type of measure, since the designer aims at minimizing the WIP level without starving the bottleneck machines. *WIP1* and *Rsys1* are dependent according to Little's law (e.g. see Cassandras 1993). Thus, one expects that a 'successful' design solution with respect to one measure is also 'successful' with respect to the other.
(3) *Average flow time for process* 2 (*Rsys*2), which is the average time that part type 3 spends in the system. *Rsys2* is a smaller-the-better (STB) type of measure. It is assumed that *Rsys2* is independent of *WIP1* and *Rsys1* since both processes are executed in different time slots.

4.2. *Simulation aspects*

Performance measures are investigated through simulations. The logic structure for the simulation model is illustrated in figure 3. The simulation model is written using the SIMAN V simulation language. The model consists of more than 100 blocks, more than 2000 entities, and is executed for a horizon of 2000 time units. In order to lower the possibility of correlated simulation runs, different seeds and streams are used for different executions. The probability distributions and parameters, which are used in the simulation, are specified in table 2.

4.3. *Applying the PSM algorithm*

The characteristics of the design problem are as follows; consider eight factors $(K=8)$, each of which has two levels $(Q=2)$. Functional requirements are given by $T=\{t_1=(0,235), t_2=(0,40), t_3=(0,135)\}$, where t_1, t_2 and t_3 denote the required

Activities	Part-type 1 distributions	Part-type 2 distributions	Part-type 3 distributions
Inter-arrival time	Normal† (8,1–2)	Normal† (8,1–2)	Normal† (10,1–2)
Load time at Robot 1	Exp. (1.2)	Exp. (1.5)	Exp. (1.1)
Unload time at Robot 1	Exp. (2.0)	Exp. (1.2)	Exp. (1.1)
Processing time at TMC1	Exp. (3.5)	—	Exp. (3.7)
Processing time at DYNA	—	Exp. (4.0)	Exp. (4.2)
Load time at Robot 2	Exp. (0.7)	Exp. (1.0)	Exp. (1.0)
Unload time at Robot 2	Exp. (2.5)	Exp. (0.7)	Exp. (0.7)
Inspection time	Exp. (2.0)	Exp. (3.0)	Exp. (2.5)
Rework time at TMC2	Exp. (5.5/6.5)	Exp. (5.5/6.5)	Exp. (5.7)
Assembling time at Robot 2	Exp. (4.5)	Exp. (4.5)	—

†Normal truncated distribution (avoiding negative values).

Table 2. The probability distributions and parameters used in the simulation model.

Columns Exp. Run	A CS	B RM	C R2	D ARR	E PR2	F DY	G PR1	H R1	*Rsys1* responses
1	+	+	+	+	+	+	+	+	276.21
2	−	+	+	+	+	−	−	−	255.72
3	+	−	+	+	−	+	−	−	258.86
4	−	−	+	+	−	−	+	+	275.78
5	+	+	−	+	−	−	−	+	245.04
6	−	+	−	+	−	+	+	−	232.92
7	+	−	−	+	+	−	+	−	232.29
8	−	−	−	+	+	+	−	+	250.12
9	+	+	+	−	−	−	+	−	262.86
10	−	+	+	−	−	+	−	+	288.36
11	+	−	+	−	+	−	−	+	290.49
12	−	−	+	−	+	+	+	−	268.20
13	+	+	−	−	+	+	−	−	243.17
14	−	+	−	−	+	−	+	+	263.27
15	+	−	−	−	−	+	+	+	262.50
16	−	−	−	−	−	−	−	−	243.90

Table 3. The 2_{IV}^{8-4} FFE (*Rsys1*).

tolerances respectively to *Rsys1*, *WIP1* and *Rsys2*. The design space includes $2^8 = 256$ feasible solutions; however, the number of experiments is constrained. In particular, a limit of 25 experiments is determined for Steps 1–6 (for each functional requirement) and a limit of 10 additional experiments is determined for Step 8. The linear model presented in (7) is selected as the underlying statistical model.

Step 1. Initialization

Step 1 is initiated with respect to *Rsys1*. 16 experiments are allocated for the initial screening design. A FFE with the highest possible resolution for a given number of factors and experiments is selected (e.g. see Montgomery 1997, Ben-Gal and Levitin 1998). Based on the underlying linear statistical model, the designer constructs the 2_{IV}^{8-4} FFE shown in table 3, such that all the main factors are not aliased with presumed two-factor interactions. The 2_{IV}^{8-4} generators are; $E = \pm BCD$; $F = \pm ACD$; $G = \pm ABC$ and $H = \pm ABD$. Full alias relationships are provided in Montgomery (1997).

Step 2. First experimentation (screening)

Each treatment in the design matrix **X** is simulated for a single run. The resulting responses are presented in table 3.

Step 3. Analysis

Factors R1, R2 and ARR are found to have significant location effects, as indicated by a normal probability plot. The estimates of the parameters of the linear statistical model that correspond to these factors are generated by the *Design Expert* statistical software (see also Montgomery 1997) and are presented in table 4. Analysis of variance, student residual plots and t-tests corroborate to a greater extent the underlying linear statistical model.

Dispersion effects are explored by: (1) plotting the residuals versus factor-levels (e.g. see figure 4); and (2) computing the dispersion statistics $F_{x_i}^*$ following (8), as seen in table 5. Factors CS, DY, and PR2 are found to have significant dispersion effects (the relatively high value of F_{RM}^* can be explained by the alias effects $\mathrm{RM} = \mathrm{CS} \cdot \mathrm{DY} \cdot \mathrm{PR2}$, i.e. $F_{\mathrm{RM}}^* = F_{\mathrm{CS \cdot DY \cdot PR2}}^*$).

Analysis of Rsys1

SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	F VALUE	PROB > F
MODEL	4636.0138	3	1545.34	230.06	< 0.0001
RESIDUAL	80.6050	12	6.72		
COR TOTAL	4716.6188	15			

ROOT MSE	2.5917	R-SQUARED	0.97
DEP MEAN	259.3547		

Predicted Residual Sum of Squares (PRESS) = 143.30

FACTOR	COEFFICIENT ESTIMATE	DF	STANDARD ERROR	t FOR H0 COEFFICIENT= 0	PROB > |t|
INTERCEPT	259.354 70	1	0.647 93		
R2	−12.706 41	1	0.647 93	−19.61	< 0.0001
ARR	5.987 11	1	0.647 93	9.24	< 0.0001
R1	−9.615 22	1	0.647 93	−14.84	< 0.0001

Table 4. Analysis of variance and t-tests for *Rsys1*.

Factors	$S_i(+)$	$S_i(-)$	F_i^*
CS	3.22	0.84	−2.68
RM	2.87	1.16	−1.79
R2	2.56	2.21	−0.29
ARR	2.36	2.43	0.05
PR2	1.12	3.08	2.02
DY	0.96	3.08	2.33
PR1	2.31	2.47	0.13
R1	2.30	2.48	0.14

Note that $F^*_{RM} = F^*_{CS \cdot DY \cdot PR2}$.

Table 5. The dispersion effect statistics for different factor levels with respect to *Rsys1*, where $F_i^* \equiv F_{x_i}^*$, $S_i(+) \equiv S[x_i(+)]$ and $S_i(-) \equiv S[x_i(-)]$.

Following the above analysis, factors are partitioned as follows: $\mathbf{x}_a = \{R2, R1, ARR\}$; $\mathbf{x}_d = \{CS, DY, PR2\}$; and $\mathbf{x}_0 = \{PR1, RM\}$.

Step 4. Fixing location and non-significant factors

Since $\mathbf{x}_a$ and $\mathbf{x}_d$ are disjoint, there is no need to calculate explicitly the *success probability* measures for factor-levels in order to fix the factors to their best levels. Therefore, the designer applies the following procedure: (1) a location factor is fixed to the level which provides the lowest location effect; (2) a dispersion factor is fixed to the level which has the lowest residual standard deviation; and (3) non-significant factors are fixed to their best (low-cost) economic level. Accordingly, the best design is estimated to be: {CS(−), RM(+), R2(−), ARR(+), PR2(+), DY(+), PR1(−), R1(−)}.

To illustrate the case where a factor x_i has both location and dispersion effects (i.e. $x_i \in \mathbf{x}_a \cap \mathbf{x}_d$), let us consider the situation where R1 $\in \mathbf{x}_a \cap \mathbf{x}_d$. According to the PSM algorithm, R1 has to be fixed to the level that yields the highest *success probability*, as shown in (10). Based on the location effects (table 4), and the dispersion

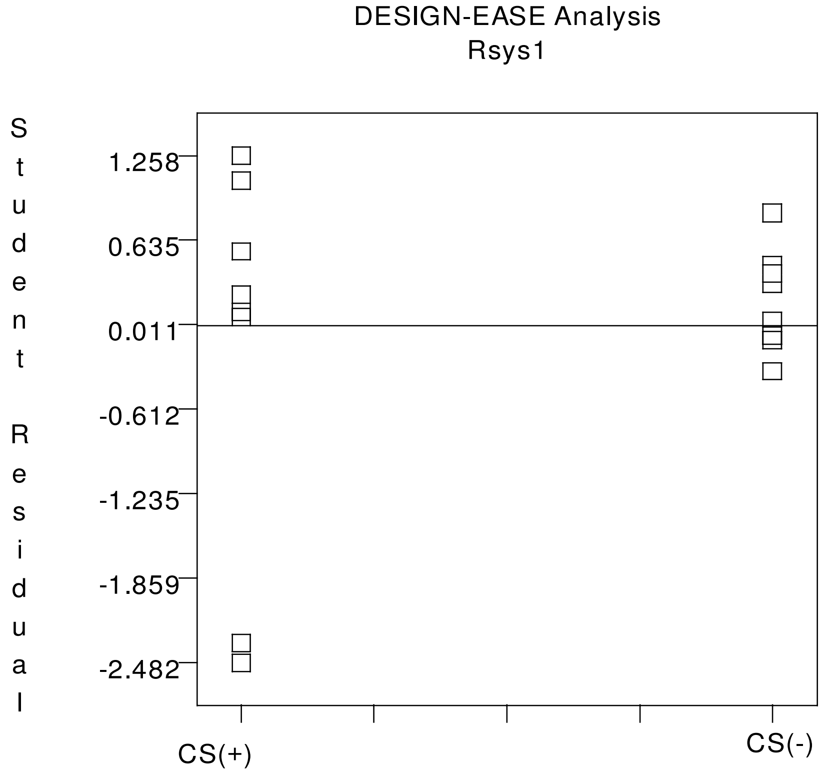

Figure 4. Residual plots versus CS(+) and CS(−) for *Rsys1*.

effects (table 5), the designer computes the *success probability* measure of each level of R1 as follows:

$$\hat{P}[R_1(+)] = \text{Prob}\{\hat{y}[R_1(+)] \le 235\} = \Phi\left(\frac{135 - 250.3}{2.30}\right) \approx 0\% \tag{11}$$

$$\hat{P}[R_1(-)] = \text{Prob}\{\hat{y}[R_1(-)] \le 235\} = \Phi\left(\frac{135 - 231.1}{2.48}\right) \approx 94.6\% , \tag{12}$$

where Φ is the standard normal distribution function, and the estimate of the mean response of R1(+) is (see table 4): $250.3 = 259.3 - 12.7 - 5.9 + 9.6$. Similarly, the estimate of the mean response of R1(−) is: $231.1 = 259.3 - 12.7 - 5.9 - 9.6$. Thus, R1(−) is the optimal level.

Step 5. Designing the second set of experiments

In order to specify the second design matrix, the designer has to find whether dispersion factors interact with one another. Figure 5 plots the residual average and the response range for each combination of the levels of CS, DY and PR2. Since any change (along an axis of the cube in figure 5) of the residual average and the response range appears to be related to the positioning on the other two axes, the designer concludes that dispersion factors do interact. Thus, a full factorial experiment at the subsequent stage is desired. The designer decides to conduct a 2^3 full factorial experi-

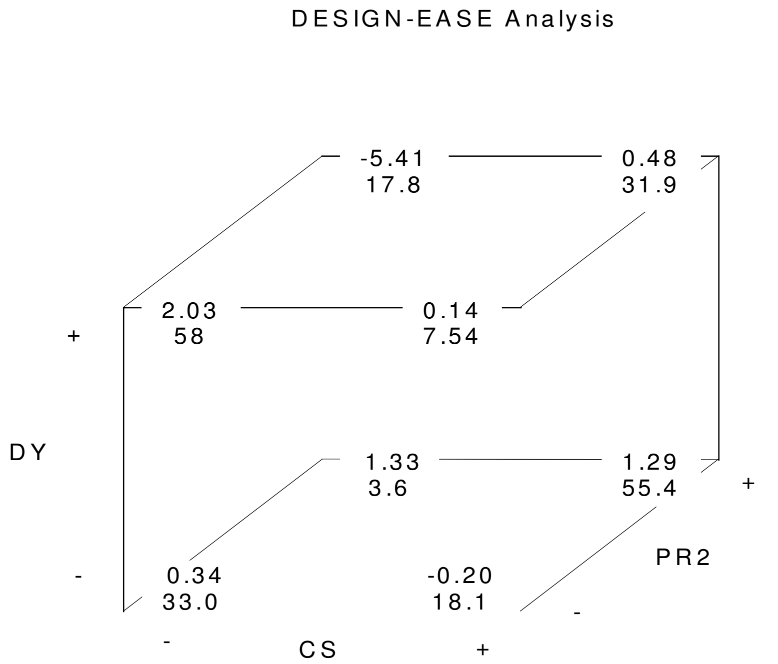

Figure 5. Residual average and response ranges with respect to *Rsys1*.

ment, since the number of remaining experiments is large enough $\left(N' \leq 9\right)$ to support such a procedure.

It is of interest to consider also the case where N' is *not* large enough to support a full factorial experiment (say, $N' = 5$). Then, the PSM suggests (section 3.2 Step 5) heuristically constructing a set of N' designs that have the highest estimated *success probability* (the set of 'most probable designs'). To illustrate how to compute the *success probability* measures in this case, let us consider three possible scenarios.

(1) **Adding replications**. Since the dispersion factors $\{\mathrm{CS}, \mathrm{DY}, \mathrm{PR2}\}$ appear to interact with one another, it is important to explore the dispersion effect associated with their combinations. Note that the 2_{IV}^{8-4} FFE with single replication (presented in table 3) provides only two data points for each combination of the dispersion factors, which is not sufficient to support a direct estimation. One solution is to add more replications of each treatment (say, $W \geq 3$), and then to employ the approach suggested in section 3.2 (based on Box and Meyer 1986a).

(2) **First-order approximation**. The first-order approximation, proposed in this paper, estimates the residual variance of any combination of dispersion factors by the sum of individual variances associated with the constituent factors. Table 6 presents a list of designs sorted in descending order by their (approximated) standard deviations. It is expected that the designs in table 6 have an identical mean response, since the location factors were already fixed in Step 4. Hence, ordering these designs according to their standard deviations is identical to ordering them according to their *success probability* measures. It follows that for $N' = 5$, the first five designs in table 6 are selected as the set of 'most probable designs'.

	CS	DY	PR2	StdV
S(+):	3.22	0.96	1.12	(1-Order
S(−):	0.84	3.08	3.08	Approx.)
1	−	+	+	1.699
2	−	+	**−**	3.336
3	−	**−**	+	3.387
4	**+**	+	+	3.544
5	−	**−**	**−**	4.440
6	**+**	+	**−**	4.560
7	**+**	−	**+**	4.599
8	**+**	**−**	**−**	5.421

Table 6. Sorted list of the 'most probable designs' using the first-order approximation scheme to compute the response standard deviations. The bold marks represent the change in factor-levels with respect to the first 'most probable design'. The location and non-significant factors are fixed as follows: RM(+), R2(−), ARR(+), PR1(−) and R1(−).

(3) **Second-order approximation**. It is seen that the estimated standard deviations, presented in the last column of table 6 have high values. This fact indicates that the first-order approximation is not sufficiently accurate. Consequently, a second-order approximation might be considered. This is done by computing the variances of subsets of residuals, each of which is associated with a combination of *two* dispersion factors (instead of a single dispersion factor). The number of data points in each group of residuals is then four instead of eight, which reduces the estimation power of the sample variance. Moreover, the number of required calculations grows exponentially with the order of the interaction (as explained in section 3.2). In particular, the second-order approximation in this case requires evaluating the variance of twelve

$$\left(12 = \binom{3}{2} \times 2^2\right)$$

subsets of residuals, as opposed to six subsets of residuals required for the first-order approximation.

Step 6. Second experimentation

A 2^3 full factorial experiment with three replications is executed for the dispersion factors CS, DY and PR2. The resulting responses, design means, design standard deviations, and *success probability* measures are presented in table 7. The designs in the table are sorted in descending order by their *success probabilities. Success probabilities* are computed by assuming a normal distribution with mean and standard deviation as estimated by Columns 7 and 8, respectively. It is interesting to note that the first five designs in table 6 (constructed according to a first-order approximation scheme) are identical to the first five designs in table 7, although ordered in a different way. Thus, retroactively, it is seen that the first-order approximation (table 6) provides those five designs with the highest *success probability*.

CS	DY	PR2	Rep1	Rep2	Rep3	Mean	STD	Success probability P^1
−	+	−	229.85	231.07	231.65	230.86	0.915	> 99.9%
+	+	+	233.99	233.31	232.65	233.31	0.669	99.4%
−	+	+	232.66	233.89	234.09	233.55	0.776	96.9%
−	−	+	229.91	235.37	231.38	232.22	2.824	83.8%
+	+	−	233.72	233.83	235.51	234.35	1.002	74.1%
−	−	−	235.16	235.91	230.12	233.73	3.147	65.7%
+	−	−	235.80	234.42	235.20	235.14	0.692	42.2%
+	−	+	238.01	241.46	240.19	239.89	1.747	0.26%

Table 7. Full factorial experiment (with three replications) for the dispersion factors CS, DY and PR2 with respect to *Rsys1*. The location and non-significant factors are fixed as follows: R1(−), R2(−), ARR(+) PR1(−) and RM(+). The last column is computed by assuming that *Rsys1* has a normal distribution with parameters estimated by the sample mean and standard deviation. The tolerance associated with *Rsys1* is $t_1 = (0, 235$ time units).

DY	PR2	RM	Rep1	Rep2	Rep3	Mean	STD	Success probability P^3
+	+	−	123.63	120.84	123.50	122.66	1.57	> 99.9%
+	−	−	122.69	123.43	123.62	123.25	0.49	> 99.9%
+	−	+	123.63	123.75	122.78	123.39	0.53	> 99.9%
−	+	+	122.43	125.68	123.51	123.87	1.65	> 99.9%
+	+	+	124.47	123.52	125.18	124.39	0.83	> 99.9%
−	−	+	126.08	125.23	126.13	125.82	0.51	> 99.9%
−	−	−	126.01	126.84	125.93	126.26	0.50	> 99.9%
−	+	−	127.64	127.19	126.26	127.03	0.70	> 99.9%

Table 8. Full factorial experiment (with three replications) for the dispersion factors: RM, DY and PR2 with respect to *Rsys2*. The location and non-significant factors are fixed as follows: PR1(+), R2(+), R1(+), ARR(+) and CS(−). The last column is computed by assuming that *Rsys2* has a normal distribution with parameters estimated by the sample mean and standard deviation. The tolerance associated with *Rsys2* is $t_3 = (0, 135$ time units).

Step 7. Repeating Steps 1–6 for the remaining functional requirements

Sets of 'most probable designs' with respect to the remaining functional requirements, t_2 and t_3 (associated respectively with the performance measures *WIP1* and *Rsys2*) , are generated by repeating Steps 1–6.

It is found that the partition of the factors to location, dispersion and non-significant categories with respect to *WIP1* is identical to the partition achieved with respect to *Rsys1*. Moreover, *Rsys1* and *WIP1* share the same the set of 'most probable designs' as well as the 'best' design solution. These phenomena are consistent with Little's law, as discussed above.

For the independent performance measure *Rsys2*, the designer obtains a different partition to location, dispersion and non-significant factors. A different set of 'most probable designs' is established, as presented in table 8. Location factors are fond to be: (1) PR1 (fixed to PR1(+) with an effect of −7.85 time units); (2) R2 (fixed to

R2(+) with an effect of −2.15 time units); and (3) R1 (fixed to R1(+) with an effect of −3.57 time units). No-significant factors are found to be: (1) ARR (fixed to ARR(+) level); and (2) CS (fixed to CS(−) level). Dispersion factors (that seem to interact one with the other) are: (1) DY, (2) PR2 and (3) RM.

Step 8. Selecting the best design

As mentioned in section 3.2, the *mega-set* of designs is obtained by merging the sets of 'most probable designs' (corresponding to *independent* functional requirements); excluding those design points in the mega-set that are expected to have a low *overall success probability*; experimenting each design point in the *filtered mega-set* with respect to the remaining functional requirement; and selecting the best design solution d^* that yields the maximum *overall success probability* by applying (4).

Following the foregoing procedure, the designer constructs the mega-set of 'most probable designs' with respect to the independent performance measures *Rsys1* and *Rsys2* as follows.

(1) The sets of 'most probable designs', as given in table 7 and table 8, are merged to obtain 16 design points.
(2) It is expected that the design points in table 8 ('best designs' with respect to *Rsys2*) will generate low *success probability* measures with respect to *Rsys1* and consequently a nearly-zero *overall success probability*. This is due to the fact that R1(+) and R2(+) increase the flow time of process 1 by +9.6 and +12.7 time units, respectively. On the other hand, the location effects of levels R1(−) and R2(−) with respect to *Rsys2* are less significant, since the upper bound of *Rsys2* (given by the tolerance t_3) is sufficiently high to tolerate such a growth (i.e. see the high values in the last column of table 8). Therefore, the designer excludes from the mega-set those designs that are included in table 8, leaving eight designs in the filtered mega-set, as presented in table 9. Note that (*a*) dispersion factors (CS, DY, PR2) and location factors (R2, R1, ARR), with respect to *Rsys1*, are set to different levels as specified in table 7; (*b*) if a non-significant factor with respect to *Rsys1* is a dispersion factor with respect to *Rsys2* (e.g. RM), then it is fixed to its best level with respect to *Rsys2*; (*c*) if a non-significant factor with respect to *Rsys1* is a location factor

Design order	A CS	B RM	C R2	D ARR	E PR2	F DY	G PR1	H R1	P^1 (Rsys1)	P^3 (Rsys2)	P^1P^3
1	+	−	−	+	+	+	−/+	−	99.4%	94.9%	94.3%
2	−	−	−	+	−	+	−/+	−	99.99%	92.5%	92.5%
3	−	−	−	+	+	+	−/+	−	96.9%	91.4%	88.6%
4	+	−	−	+	−	+	−/+	−	74.1%	98.8%	73.2%
5	−	−	−	+	+	−	−/+	−	83.8%	84.7%	71.0%
6	−	−	−	+	−	−	−/+	−	65.7%	77.6%	50.9%
7	+	−	−	+	−	−	−/+	−	42.2%	62.8%	26.5%
8	+	−	−	+	+	−	−/+	−	0.26%	99.0%	0.26%

Table 9. The final set of 'most probable designs'. The *overall success probability* measures with respect to both *Rsys1* (with tolerance t_1) and *Rsys2* (with tolerance t_3) are provided in the last column. PR1 is an online-adjustable factor of the FMC, therefore, it is fixed to the (−) level in the first process and to the (+) level in the second process.

with respect to *Rsys2* (e.g. PR1), then it is fixed to its best level with respect to *Rsys2* (note, however, that PR1 is an online-adjustable factor of the FMC and hence can be fixed to its $(-)$ level during the first process); and (*d*) if a factor is non-significant with respect to both *Rsys1* and *Rsys2*, then it is fixed according to its best economic level.

(3) The designer conducts eight additional experiments (each with three replications) in order to estimate the success probability measures (with respect to *Rsys2*) of the constituent designs in table 9. These estimates are used to compute the overall success probability measures as presented in the last column of table 9. Finally, the designer selects the design point that yields the highest overall success probability, $d^* = \{\mathrm{CS}(+), \mathrm{RM}(-), \mathrm{R2}(-), \mathrm{ARR}(+), \mathrm{PR2}(+), \mathrm{PR1}(-/+), \mathrm{DY}(+), \mathrm{R1}(-)\}$, with an overall success probability measure of 94.3%, although it is not the most successful design with respect to each of the individual requirements. The remaining designs are sorted in descending order by their overall success probabilities.

5. Summary

In Suh (1990, 1995) a functional complexity measure was provided as a rational means for quantifying how well a proposed design satisfies the governing requirements. In this paper, the functional complexity measure is shown also to have a heuristic merit. The proposed PSM approach provides a detailed design paradigm, although further research is required to support its applicability to wide range of design problems. The main contribution of the PSM lies in its underlying simplicity and its modular structure which includes: (1) screening and estimating the main effects to attain a tractable problem size; (2) generating probabilistic measures to determine designs that maximize the likelihood of satisfying a given set of functional requirements; and (3) applying a heuristic search to select the best design solution. Each of these steps can be modified by the designer in several respects to fit the specific design problem better. Some examples are: (1) replacing the approximation method used to derive the dispersion effects by the exact method suggested by Box and Meyer (1986a); (2) extending the underlying statistical model to include higher-order interactions; (3) using different procedures to derive the probabilistic measures; and (4) applying other heuristics in order to construct the sets of 'most probable designs'. All of these modifications, however, maintain the general notion of a probabilistic design within a stochastic framework. Moreover, the PSM can be modified to support redesign activities by a computerized database, as suggested in Ben-Gal *et al.* (1997).

Further research is required to determine policies for resource allocation in sequential experiments and how to consider a set of *dependent* functional requirements. Ben-Gal *et al.* (1997) suggest approaching this problem by introducing the concept of *conditional* success probability measures.

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